Engineering Analytic Principles and Predictive Computational Skills for K-12 Students:

Presenting a List of High School 9th Grade Age-Possible Statics Topics to Engineering and Technology Educators and Curriculum Developers

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Introduction

Rationale for Presenting this List

Problems in the current K-12 engineering curriculum

In the most recent decade, high schools across the United States have tried to incorporate engineering design into traditional technology curriculum, with various degrees of success. Smith and Wicklein (2007, pp. 2-3) affirmed the achievements made so far in “the integration of engineering design into secondary technology education classes,” but also indicated that the “fragmented focus and lack of a clear curriculum framework” had been “detrimental to the potential of the field and have hindered efforts aimed at achieving the stated goals of technological literacy for all students.” An authoritative report issued on September 8, 2009, by the Committee on K-12 Engineering Education established by the National Academy of Engineering and the National Research Council, titled *Engineering in K-12 Education: Understanding the Status and Improving the Prospects* (2009), confirmed the existence of similar problems in the current K-12 engineering education; one of the most serious problems is the absence of a cohesive K-12 engineering curriculum; “engineering design, the central activity of engineering, is predominant in most K-12 curricular and professional development programs. The treatment of key ideas in engineering, many closely related to engineering design, is much more uneven;” in addition, the Committee on K-12 Engineering Education commented on the “absence of a clear description of which engineering knowledge, skills, and habits of mind are most important, how they relate to and build on one another, and how and when (i.e., at what age) they should be introduced to students” (2009, pp. 7-8; p. 151). Unlike mathematics, chemistry and physics, K-12 engineering curriculum remains skeletal so far in American K-12 system; its main focus is on generic design process; and currently, analytic and predictive knowledge contents covered in most K-12 engineering curriculum in the United States are restricted to a few areas (such as CAD, electronics, and robotics), and are generally not sequentially organized.

Research questions and practical conceptual framework

The above evaluation of the current status of K-12 engineering education could inspire us to ask these questions: (1) How could we determine what engineering analytic principles and predictive skills from what subject should be taught to students at what Grade in the K-12 curriculum, in a rational and scientific way? (2) How could we make sure that what students learned at high schools in engineering curriculum could be transferred to university programs?

It appears that no solid research has been conducted to establish a practical instrument for the solution of the above fundamental issues. However, based on the way engineering curriculum has been historically established, I have constructed a practical conceptual framework to answer the above two questions, during the Spring Semester, 2009, as a Graduate Research Assistant at the University of Georgia. If we read any typical information sheet for university level undergraduate engineering program, we will see that the courses are organized in a sequence based on the fulfillment of pre-requisites in mathematics, physics, chemistry, technology and previous engineering courses; and this pre-requisite sequence is usually listed in course descriptions. Therefore, for all practical purposes, we could hypothesize that the same principles used historically in the establishment of curricular structure in university undergraduate engineering programs could apply to the rational and scientific determination of K-12 age-possible engineering analytic principles and predictive skills at any particular Grade, and for any particular subject of engineering; in addition, based on the fact that university undergraduate engineering textbooks, especially those used in foundation courses (such as statics, dynamics, strength of materials, etc.), all contain portions that are based on pre-calculus mathematics and scientific principles which are usually covered in K-12 mathematics and science curriculum, we could also hypothesize that these pre-calculus portions of engineering topics could possibly be taught at various Grade levels, provided that the pre-requisite pre-calculus mathematics and science principles have been covered in previous Grade levels, according to the performance standards in mathematics and science established by any particular state. This hypothetic conceptual framework could constitute a practical tool to be used in the solution of the problem of defining K-12 age-possible engineering analytic knowledge content.

On Friday March 27, 2009, during the International Technology Education Association's 71st Annual Conference held in Kentucky International Convention Center in Louisville, Kentucky, and under the sponsorship of Dr. John Mativo, from the University of Georgia, I have presented a *Proposed Model for a Clear Description of K-12 Age-Possible Engineering Knowledge Content*; under this Proposed Model, Selection of K-12 age-possible engineering analytic principles and predictive skills for various Grade levels should be based on the mastery of mathematics and science (notably physics and chemistry) pre-requisites, as mandated by national or state performance standards for previous or same Grade levels. During the Spring Semester 2009, as a Graduate Research Assistant at the University of Georgia, I used the following steps to construct tables of high school (9th Grade) age-possible statics and fluid mechanics topics with analytic principles and computational formulas:

(1) Select textbooks and instructor's solution manuals that are among the most popular ones for undergraduate engineering statics and fluid mechanics courses;

(2) Read carefully every paragraph in the body text to find and record the pre-requisite science knowledge content needed for each topic (notably physics and chemistry). Practically speaking, every engineering topic includes engineering principles that are based on concepts of physics and/or chemistry, which shall reveal the knowledge of physics and/or chemistry required for students to comfortably learn the topic by thoroughly understanding the underlying scientific principles. For example, the pressure is defined as force exerted per unit area, or $\vec{P} \equiv \frac{\vec{F}}{A}$ where $\vec{P}$ is the pressure and $\vec{F}$ is the force exerted on a surface area A; and the force $\vec{F}$ is defined as mass m multiplied by acceleration $\vec{a}$ in Newton's First Law, or $\vec{F} \equiv m\vec{a}$; thus, Newton's First Law is the pre-requisite principle for high school students to master before the

concept of pressure could be comfortably learned. If we want to teach the concept of pressure to students at 9th Grade, then Newton's First Law must either be taught at 8th Grade, or still at 9th Grade but before the concept of pressure is taught, in a correct pre-requisite sequence.

(3) Find the relevant computational formulas to determine and record the mathematics skills needed. Practically speaking, every engineering topic includes mathematically-based formulas or equations, which shall reveal the level of mathematics required for students to comfortably learn the topic's analytic principles and formula-based predictive computational skills. For example, the formula for the calculation of pressure is $\vec{P} \equiv \frac{\vec{F}}{A}$, where $\vec{P}$ is the pressure, $\vec{F}$ is the force exerted on a surface area A; this formula involves division and multiplication as well as calculation of surface area; thus, mathematically speaking, it could be taught only after students master computational skills related to division and multiplication as well as calculation of surface area taught in geometry.

(4) Compare the recorded data, i.e., mathematics and science pre-requisites, with the mandates of the Performance Standards for Mathematics and Sciences of the Department of Education of a selected state in the Southern part of the United States, to determine the Grade level for the inclusion of the topic.

Objective of Presenting this List

This *List* is intended to be an "initial list" of high school 9th Grade "age-possible" statics topics; whether these topics are actually age-feasible or age-appropriate could be determined only after actual pedagogic experiment or pilot studies have been conducted and analyzed. However, the presentation of this *List* could constitute the critical first step for the extensive integration of statics-related engineering analytic principles and predictive computational skills into a viable K-12 engineering and technology curriculum, in a rational, systemic and cohesive way.

Hopefully, the presentation of this *List* could help improve engineering education in the United States, with the following practical applications:

(1) <u>K-12 engineering curriculum development</u>: Current K-12 engineering and technology curriculum developers and teachers, in their endeavors to integrate engineering analytic principles and predictive skills into K-12 engineering and technology curriculum, in a cohesive and systematic way, could use this *List* as a reference in the selection of statics topics from the main textbook listed in Table 1, for pedagogic experiment or pilot study aimed at determining if the topics included in the *List* are indeed age-feasible or age-appropriate for high school 9th Grade students.

(2) <u>Engineering education</u>: K-12 engineering teachers as well as university undergraduate engineering professors could use the *List* as a reference to review pertinent mathematics skills and scientific principles at the start of engineering courses with their students, for the statics topics that require only pre-calculus mathematics skills.

(3) K-12 mathematics and science education: K-12 teachers could use this *List* as a reference to create extra learning materials focused on the applications of mathematics skills and scientific principles in engineering, and thus, help students to understand the relevance of mathematics skills and scientific principles to practical solution of engineering design problems.

Source of Data

University undergraduate statics textbook and pertinent instructional materials that have been used as data source in the research are shown in Table 1 and *Figure 1*.

Table 1. Textbook Information

	Main Textbook	**Instructor's Solution Manuals**	
Title	Vector Mechanics for Engineers Statics, 7th Edition	Instructor's and Solutions Manual to Accompany Vector Mechanics for Engineers – Statics, 7th Edition, Volume 1	Instructor's and Solutions Manual to Accompany Vector Mechanics for Engineers – Statics, 7th Edition, Volume 2
Authors	Ferdinand P. Beer & E. Russell Johnston & Elliot R. Eisenberg	Ferdinand P. Beer & E. Russell Johnston & Elliot R. Eisenberg	Ferdinand P. Beer & E. Russell Johnston & Elliot R. Eisenberg
Publisher	McGraw-Hill Higher Education	McGraw-Hill Higher Education	McGraw-Hill Higher Education
Year	2004	2004	2004
ISBN	0-07-230493-6	10: 0072536055	10: 0072962623
Application	Used for the extraction of statics related engineering analytic/predictive principles and computational formulas.	Used to double-check for the mathematics and physics principles and computational skills needed for the study of various topics of statics contained in the main textbook.	

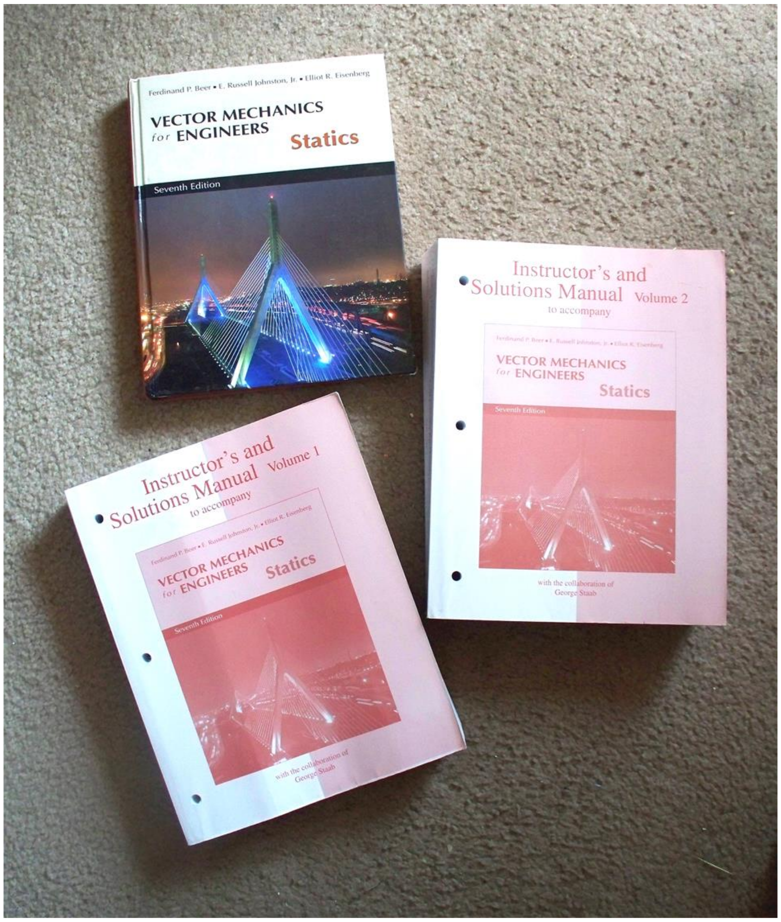

Figure 1. University undergraduate statics textbook and Instructor's Solution Manuals used in the research as data source (used at California State University, Los Angeles).

Outcomes of the Research

The outcome of this research is very encouraging. A substantial amount of engineering declarative knowledge content covered in the selected university undergraduate statics textbook has been initially determined to be pedagogically possible for 9th Grade high school students, based on the mandates of the Mathematics and Science Performance Standards of a selected state in the United States.

Initial Determination of High School Age-Possible Statics Topics

Table 2 constitutes the *Initial List of High School 9th Grade Age-Possible Statics Topics*, and is the centerpiece of this article. *Figure 2* illustrate how to use this *List*.

The statistic summary of the research project (Table 4) indicates that a significant portion of statics knowledge content covered in the selected undergraduate level textbook could possibly be taught to high school students at 9th Grade. 58.7% of all sections, and 56.0% of the volume in the selected textbook is based on pre-calculus mathematics and on principles of physics students are supposed to learn before or by 9th Grade, according to the Mathematics and Science Performance Standards of the selected state's Department of Education.

Initial Determination of Pre-Requisite Mathematics and Science Topics

Table 3 constitutes the *Pre-Requisite Mathematics and Science Topics to Be Reviewed Before Teaching the Pre-Calculus Portion of Statics Topics to 9th Grade Students*. This list includes 17 sets of mathematics principles and skills, as well as 7 sets of physics principles and skills that are needed as pre-requisites or as important topics to be reviewed for the effective learning of statics topics initially determined as appropriate for 9th Grade students.

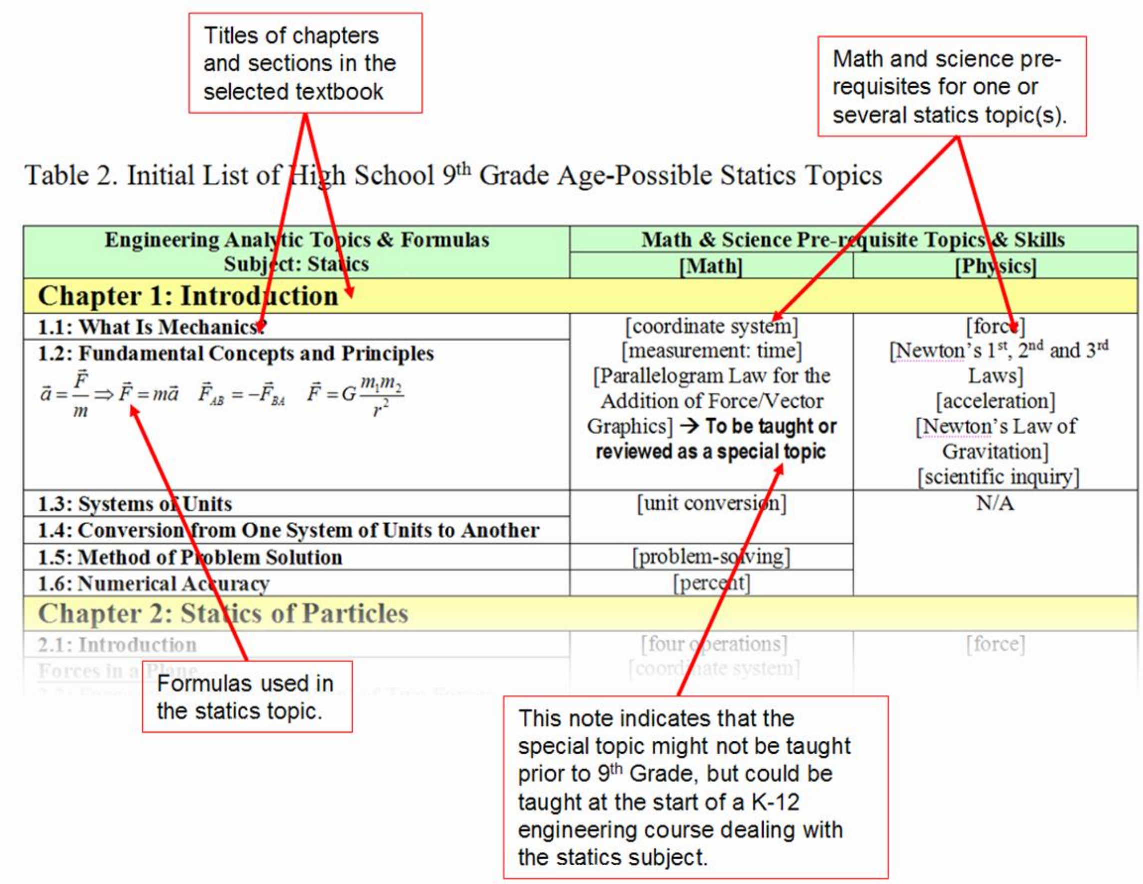

Table 2. Initial List of High School 9th Grade Age-Possible Statics Topics

Engineering Analytic Topics & Formulas Subject: Statics	Math & Science Pre-requisite Topics & Skills [Math]	[Physics]
Chapter 1: Introduction		
1.1: What Is Mechanics?	[coordinate system] [measurement: time] [Parallelogram Law for the Addition of Force/Vector Graphics] → **To be taught or reviewed as a special topic**	[force] [Newton's 1st, 2nd and 3rd Laws] [acceleration] [Newton's Law of Gravitation] [scientific inquiry]
1.2: Fundamental Concepts and Principles $\vec{a}=\frac{\vec{F}}{m}\Rightarrow\vec{F}=m\vec{a}$ $\vec{F}_{AB}=-\vec{F}_{BA}$ $\vec{F}=G\frac{m_1m_2}{r^2}$		
1.3: Systems of Units	[unit conversion]	N/A
1.4: Conversion from One System of Units to Another		
1.5: Method of Problem Solution	[problem-solving]	
1.6: Numerical Accuracy	[percent]	
Chapter 2: Statics of Particles		
2.1: Introduction	[four operations]	[force]
Forces in a Plane	[coordinate system]	

Figure 2. The Initial List of High School 9th Grade Age-Possible Statics Topics.

Conclusions and Recommendations

This article has provided (1) a reference list for high school 9th Grade age-possible statics topic, and (2) a reference list for the review of mathematics and science pre-requisites. In order to improve K-12 engineering education, the following recommendations and plans are hereby presented for consideration, support and implementation:

1. Research: I shall continue research on defining K-12 age-possible engineering knowledge content from the subjects of (1) dynamics, (2) strength of

materials, (3) material science, (4) heat transfer, (5) thermodynamics, (6) engineering economics, (7) aerodynamics and (8) mechanism design, using popular university undergraduate engineering textbooks and auxiliary instructional materials (such as instructor's or student's solution manual) as sources of data, and the mandates of Mathematics and Science Performance Standards of the selected state in the United States as guidelines; this will lead to the eventual publication of *The Handbook of Proposed Engineering Topics with Analytic Principles, Computational Formulas and Units for K-12 Schools (with Reviews for Mathematics and Sciences)*. This research could constitute the most important pre-requisite for the implementation of a viable K-12 engineering curriculum with a cohesive and systemic sequence of knowledge content. It would be an important reference for (1) the development of K-12 engineering teaching materials, and (2) the improvement of K-12 engineering and technology teacher training programs.

2. Pilot study: K-12 schools (especially high schools, including chartered high schools) could be found to conduct pilot pedagogic experiments to determine the age-feasibility and age-appropriateness of all statics-related analytic knowledge content identified in *Initial List of High School 9th Grade Age-Possible Statics Topics*. Likewise, K-12 mathematics and science teachers could use the same *List* as a reference to incorporate pertinent statics topics in their respective curriculum.
3. Change within the system: We could encourage existing K-12 engineering and technology curriculum developers to use the same *List* as a reference to incorporate statics-related engineering knowledge and skills into their previously developed instructional materials, or to create new ones.

References

Committee on K-12 Engineering Education (2009). *Engineering in K-12 education: Understanding the status and improving the prospects*. Washington, DC: National Academy of Engineering and the National Research Council.

Smith, P. C., & Wicklein, R. C. (2007). *Identifying the essential aspects and related academic concepts of an engineering design curriculum in secondary technology education.* Unpublished internal research report, NCETE. Retrieved January 30, 2009 from http://ncete.org/flash/publications.php

Table 2. Initial List of High School 9th Grade Age-Possible Statics Topics

Engineering Analytic Topics & Formulas Subject: Statics	Math & Science Pre-requisite Topics & Skills [Math]	 [Physics]
Chapter 1: Introduction		
1.1: What Is Mechanics?	[coordinate system] [measurement: time] [Parallelogram Law for the Addition of Force/Vector Graphics] → **To be taught or reviewed as a special topic**	[force] [Newton's 1st, 2nd and 3rd Laws] [acceleration] [Newton's Law of Gravitation] [scientific inquiry]
1.2: Fundamental Concepts and Principles $\vec{a} = \frac{\vec{F}}{m} \Rightarrow \vec{F} = m\vec{a}$ $\vec{F}_{AB} = -\vec{F}_{BA}$ $\vec{F} = G\frac{m_1 m_2}{r^2}$		
1.3: Systems of Units	[unit conversion]	N/A
1.4: Conversion from One System of Units to Another		
1.5: Method of Problem Solution	[problem-solving]	
1.6: Numerical Accuracy	[percent]	
Chapter 2: Statics of Particles		
2.1: Introduction	[four operations] [coordinate system]	[force]
<u>Forces in a Plane</u> **2.2: Force on a Particle. Resultant of Two Forces**		
2.3: Vectors	[vector graphics] → **To be taught or reviewed as a special math topic**	
2.4: Addition of Vectors		
2.5: Resultant of Several Concurrent Forces		
2.6: Resolution of a Force into Components	[vector graphics] [trigonometric functions]	
2.7: Rectangular Components of a Force. Unit Vectors		
2.8: Addition of Forces by Summing *x* and *y* Components $\vec{F} = F_x\hat{i} + F_y\hat{j}$ $F_x = F\cos\theta$ $F_y = F\sin\theta$ $\tan\theta = \frac{F_y}{F_x}$ $F = \sqrt{F_x^2 + F_y^2}$	[four operations] [square root] [trigonometric functions] → To be taught or reviewed as a special math topic [coordinate system]	
2.9: Equilibrium of a Particle $R = \sum F = F_1 + F_2 + ... = 0 \Rightarrow R_x = \sum F_x = 0$ $R_y = \sum F_y = 0$ $R_z = \sum F_z = 0$	[sigma notation] → **To be taught or reviewed as a special math topic** [four operations]	
2.10: Newton's First Law of Motion	[four operations]	[Newton's 1st, 2nd and 3rd Laws] [acceleration]
2.11: Problems Involving the Equilibrium of a Particle. Free-Body Diagrams		
<u>Forces in Space</u> **2.12: Rectangular Components of a Force in Space** $F_y = F\cos\theta_y$ $F_h = F\sin\theta_y$ $F_x = F_h\cos\phi = F\sin\theta_y\cos\phi$ $F_z = F_h\sin\phi = F\sin\theta_y\sin\phi$ $F^2 = F_y + F_h = F_y + F_x + F_z \rightarrow F = \sqrt{F_x + F_y + F_z}$ $F_x = F\cos\theta_x$ $F_y = F\cos\theta_y$ $F_z = F\cos\theta_z$ $(0^\circ < \theta_{x,y,z} < 180^\circ)$ $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ $\vec{F} = F(\cos\theta_x\hat{i} + \cos\theta_y\hat{j} + \cos\theta_z\hat{k})$ $\cos\theta_x = \frac{F_x}{F} = \frac{d_x}{d} = \frac{R_x}{R}$ $\cos\theta_y = \frac{F_y}{F} = \frac{d_y}{d} = \frac{R_y}{R}$ $\cos\theta_z = \frac{F_z}{F} = \frac{d_z}{d} = \frac{R_z}{R}$ $\theta_{x(y,z)} = \cos^{-1}\frac{F_{x(y,z)}}{F} = \cos^{-1}\frac{d_{x(y,z)}}{d}$ $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$ $\hat{\lambda} = \cos\theta_x\hat{i} + \cos\theta_y\hat{j} + \cos\theta_z\hat{k}$ $\hat{\lambda} = \frac{\vec{F}}{F}$ $\hat{i} = \frac{d_x}{d}$ $\hat{j} = \frac{d_y}{d}$ $\hat{k} = \frac{d_z}{d}$ $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1 \rightarrow \hat{\lambda}_x^2 + \hat{\lambda}_y^2 + \hat{\lambda}_z^2 = 1$	[four operations] [square root] [trigonometric functions] → **To be taught or reviewed as a special math topic** [coordinate system]	[force]

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Statics	Math & Science Pre-requisite Topics & Skills	
	[Math]	**[Physics]**
Chapter 2: Statics of Particles (Continued)		
2.13: Force Defined by Its Magnitude and Two Points on Its Line of Action $\overrightarrow{MN} = d_x\hat{i} + d_y\hat{j} + d_z\hat{k}$ $\hat{\lambda} = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d}\left(d_x\hat{i} + d_y\hat{j} + d_z\hat{k}\right)$ $d_{x(y,z)} = x(y,z)_2 - x(y,z)_1 \quad d = \sqrt{d_x^2 + d_y^2 + d_z^2}$ $\vec{F} = F\hat{\lambda} = \frac{F}{d}\left(d_x^2\hat{i} + d_y^2\hat{j} + d_z^2\hat{k}\right)$ $F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}$	[four operations] [square root] [trigonometric functions] **→ To be taught or reviewed as a special math topic** [coordinate system]	[force] [Newton's 1st, 2nd and 3rd Laws]
2.14: Addition of Concurrent Forces in Space $\vec{R} = \sum \vec{F} \quad R = \sqrt{R_x^2 + R_y^2 + R_z^2}$ $R_x\hat{i} + R_y\hat{j} + R_z\hat{k} = \left(\sum F_x\right)\hat{i} + \left(\sum F_y\right)\hat{j} + \left(\sum F_z\right)\hat{k}$		
2.15: Equilibrium of a Particle in Space $R = \sum F = F_1 + F_2 + ... = 0 \quad \rightarrow \quad R_x = \sum F_x = 0 \quad R_y = \sum F_y = 0$ $R_z = \sum F_z = 0$ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax+by+cz \\ dx+ey+fz \\ gx+hy+iz \end{bmatrix} \quad \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \times \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $R_x = \sum F_x = 0 \quad aF_1 + bF_2 + cF_3 = 0$ $R_y = \sum F_y = 0 \quad dF_1 + eF_2 + fF_3 = 0$ $R_z = \sum F_z = 0 \quad gF_1 + hF_2 + iF_3 = 0$ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \times \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} aF_1+bF_2+cF_3 \\ dF_1+eF_2+fF_3 \\ gF_1+hF_2+iF_3 \end{bmatrix}$	[coordinate system] [four operations] [linear algebra] **→ To be taught or reviewed as a special math topic**	
Chapter 3: Rigid Bodies - Equivalent Systems of Forces		
3.1: Introduction **3.2: External and Internal Forces** **3.3: Principle of Transmissibility. Equivalent Forces**	[four operations] [geometry: point, axis/line, 3D body]	[force] [motion]
3.4: Vector Product of Two Vectors $\vec{V} = \vec{P} \times \vec{Q} \quad V = PQ\sin\theta \quad \vec{V} \perp \vec{P} \quad \vec{V} \perp \vec{Q} \quad \vec{V} \perp Plane_{\vec{P},\vec{Q}}$ $\vec{P} \times \left(\vec{Q}_1 + \vec{Q}_2\right) = \vec{P} \times \vec{Q}_1 + \vec{P} \times \vec{Q}_2 \quad \left(\vec{P} \times \vec{Q}\right) \times \vec{S} \neq \vec{P} \times \left(\vec{Q} \times \vec{S}\right)$ $\vec{V} = \vec{Q} \times \vec{P} = -\left(\vec{P} \times \vec{Q}\right) \quad \vec{Q} \times \vec{P} \neq \vec{P} \times \vec{Q} \quad \vec{P} \times \vec{Q} = -\vec{V}$ $\vec{P} \times \left(\vec{Q}_1 + Q_2\right) = \vec{P} \times \vec{Q}_1 + \vec{P} \times Q_2 \quad \vec{Q} \times \vec{P} \neq \vec{P} \times \vec{Q}$ $\vec{V} = \vec{Q} \times \vec{P} = -\left(\vec{P} \times \vec{Q}\right) \quad \vec{P} \times \vec{Q} = -\vec{V} \quad \vec{V} = \vec{P} \times \vec{Q}$ $\left(\vec{P} \times \vec{Q}\right) \times \vec{S} \neq \vec{P} \times \left(\vec{Q} \times \vec{S}\right)$	[trigonometric functions] **→ To be taught or reviewed as a special math topic** [cross product] **→ To be taught or reviewed as a special math topic**	
3.5: Vector Products Expressed in Terms of Rectangular Components $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0 \quad \hat{i} \times \hat{j} = \hat{k} \quad \hat{j} \times \hat{k} = \hat{i} \quad \hat{k} \times \hat{i} = \hat{j}$ $\hat{i} \times \hat{k} = -\hat{j} \quad \hat{j} \times \hat{i} = -\hat{k} \quad \hat{k} \times \hat{j} = -\hat{i}$ $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k} \quad \vec{Q} = Q_x\hat{i} + Q_y\hat{j} + Q_z\hat{k}$ $\vec{V} = \vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$ $V_x = P_yQ_z - P_zQ_y \quad V_y = -\left(P_xQ_z - P_zQ_x\right) = P_zQ_x - P_xQ_z$ $V_z = P_xQ_y - P_yQ_x$	[trigonometric functions] [cross product] **→ To be taught or reviewed as a special math topic** [dot product] **→ To be taught or reviewed as a special math topic**	[force]

Table 2. (Continued).

Engineering Analytic Topics & Formulas Subject: Statics	Math & Science Pre-requisite Topics & Skills [Math]	 [Physics]
Chapter 3: Rigid Bodies - Equivalent Systems of Forces (Continued)		
3.6: Moment of a Force about a Point $\vec{M}_0 = \vec{r} \times \vec{F} \quad M_0 = rF\sin\theta = Fd$ $\vec{r} = \vec{v}^{O\rightarrow A}_{position} \quad \theta = \angle_{\vec{r}\rightarrow\vec{F}} \quad d \perp \vec{F}$ $\vec{M}_0 = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$ $M_x = yF_z - zF_y \quad M_y = -(xF_z - zF_x) = zF_x - xF_z$ $M_z = xF_y - yF_x$	[four operations] [geometry: point, axis/line, 3D body] [cross product] → **To be or reviewed taught as a special math topic** [dot product] → **To be taught or reviewed as a special math topic** [linear algebra] → **To be taught or reviewed as a special math topic**	[force]
3.7: Varignon's Theorem $\vec{r} \times (\vec{F}_1 + \vec{F}_2 + ...) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + ...$	[four operations] [cross product] → **To be taught or reviewed as a special math topic** [dot product] → **To be taught or reviewed as a special math topic**	
3.8: Rectangular Components of the Moment of a Force $\vec{M}_B = \vec{r}_{A/B} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix}$ $\vec{r}_{A/B} = x_{A/B}\hat{i} + y_{A/B}\hat{j} + z_{A/B}\hat{k}$ $x_{A/B} = x_A - x_B \quad y_{A/B} = y_A - y_B \quad z_{A/B} = z_A - z_B$	[four operations] [cross product] → **To be taught as a special math topic**	
3.9: Scalar Product of Two Vectors $\vec{P} \bullet \vec{Q} = PQ\cos\theta = P_xQ_x + P_yQ_y + P_zQ_z \quad \theta = \angle_{\vec{P}\rightarrow\vec{Q}}$ $\vec{P} \bullet \vec{Q} = \vec{Q} \bullet \vec{P} \quad \vec{P} \bullet (\vec{Q}_1 + \vec{Q}_2) = \vec{P} \bullet \vec{Q}_1 + \vec{P} \bullet \vec{Q}_2$ $P_{OL} = \vec{P} \bullet \hat{\lambda} = P_x\cos\theta_x + P_y\cos\theta_y + P_z\cos\theta_z$ (More formulas on p. pp. 94-95)	[four operations] [dot product] → **To be taught or reviewed as a special math topic**	
3.10: Mixed Triple Product of Three Vectors $\vec{S} \bullet (\vec{P} \times \vec{Q}) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$	[four operations] [cross product] → **To be taught or reviewed as a special math topic**	
3.11: Moment of a Force about a Given Axis $M_{OL} = \hat{\lambda} \bullet \vec{M}_O = \hat{\lambda} \bullet (\vec{r} \times \vec{F}) = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$ (More formulas on p. pp. 98)	[four operations] [dot product] → **To be taught or reviewed as a special math topic**	
3.12: Moment of a Couple $\vec{M} = \vec{r} \times \vec{F} \quad M = rF\sin\theta = Fd$	[four operations] [trigonometric functions] [cross product] → **To be taught or reviewed as a special math topic**	[force] [motion] [lever]
3.13: Equivalent Couples $F_1d_1 = F_2d_2$	[four operations] [geometry: point, axis/line, 3D body]	

Table 2. (Continued).

Engineering Analytic Topics & Formulas **Subject: Statics**	**Math & Science Pre-requisite Topics & Skills**	
	[Math]	**[Physics]**
Chapter 3: Rigid Bodies - Equivalent Systems of Forces (Continued)		
3.14: Addition of Couples $\vec{M} = \vec{r} \times \vec{R} = \vec{r} \times (\vec{F}_1 + \vec{F}_2) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 \quad \vec{M} = \vec{M}_1 + \vec{M}_2$	[four operations] [cross product] → **To be taught or reviewed as a special math topic**	[force]
3.15: Couples Can Be Represented by Vectors	[vector graphics] → **To be taught or reviewed as a special math topic**	
3.16: Resolution of a Given Force Into a Force at *O* and a Couple $\vec{M}_{O'} = \vec{r}' \times \vec{F} = (\vec{r} + \vec{s}) \times \vec{F} = \vec{r} \times \vec{F} + \vec{s} \times \vec{F}$ $\vec{M}_{O'} = \vec{M}_O + \vec{s} \times \vec{F}$	[four operations] [cross product] → **To be taught or reviewed as a special math topic**	
3.17: Reduction of a System of Forces to One Force and One Couple $\vec{R} = \sum \vec{F} \quad \vec{M}_O^R = \sum \vec{M}_O = \sum (\vec{r} \times \vec{F})$ $\vec{M}_{O'}^R = \vec{M}_O + \vec{s} \times \vec{R} \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \quad \vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$ $\vec{M}_O^R = M_x^R\hat{i} + M_y^R\hat{j} + M_z^R\hat{k}$		
3.18: Equivalent Systems of Forces $\sum \vec{F} = \sum \vec{F}' \quad \& \quad \sum \vec{M}_0 = \sum \vec{M}'_0$ $\sum \vec{F} = \sum \vec{F}' \quad and \quad \sum \vec{M}_O = \vec{M}_O'$ $\sum F_x = \sum F'_x \quad \sum F_y = \sum F'_y \quad \sum F_z = \sum F'_z$ $\sum M_x = \sum M'_x \quad \sum M_y = \sum M'_y \quad \sum M_z = \sum M'_z$	[four operations] [coordinate system]	
3.19: Equipollent Systems of Vectors	[vector graphics] → **To be taught or reviewed as a special math topic**	
3.20: Further Reduction of a System of Forces	[coordinate system]	
3.21: Reduction of a System of Forces to a Wrench $p = \frac{M_1}{R} \quad M_1 = \frac{\vec{R} \bullet \vec{M}_O^R}{R} \quad p = \frac{M_1}{R} = \frac{\vec{R} \bullet \vec{M}_O^R}{R^2}$ $\vec{M}_1 = p\vec{R} \rightarrow \begin{matrix} \vec{M}_1 + \vec{r} \times \vec{R} = \vec{M}_O^R \\ p\vec{R} + \vec{r} \times \vec{R} = \vec{M}_O^R \end{matrix}$	[four operations] [geometry: point, axis/line, 3D body] [trigonometric functions] → **To be taught or reviewed as a special math topic** [dot product] → **To be taught or reviewed as a special math topic** [cross product] → **To be taught or reviewed as a special math topic**	[force] [motion] [lever]
Chapter 4: Equilibrium of Rigid Bodies		
4.1: Introduction $\sum \vec{F} = 0 \quad \sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$ $\sum \vec{M}_O = \sum (\vec{r} \times \vec{F}) = 0 \quad \sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$	[sigma notation] → **To be taught or reviewed as a special math topic** [coordinate system]	[force] [Newton's 3^{rd} Law: Action and Reaction]
4.2: Free-Body Diagram		
<u>Equilibrium in Two Dimensions</u> **4.3: Reactions at Supports and Connections for a Two-Dimensional Structure**		

Table 2. (Continued).

Engineering Analytic Topics & Formulas **Subject: Statics**	**Math & Science Pre-requisite Topics & Skills** **[Math]**	**[Physics]**
Chapter 4: Equilibrium of Rigid Bodies (Continued)		
4.4: Equilibrium of a Rigid Body in Two Dimensions $F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$ $\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_O = 0$ $\sum M_A = 0 \quad \sum M_B = 0 \quad \sum M_C = 0$	[sigma notation] → **To be taught or reviewed as a special math topic** [coordinate system]	[force] [Newton's 3rd Law: Action and Reaction]
4.5: Statically Indeterminate Reactions. Partial Constraints		
4.6: Equilibrium of a Two-Force Body		
4.7: Equilibrium of a Three-Force Body		
<u>Equilibrium in Three Dimensions</u> **4.8: Equilibrium of a Rigid Body in Three Dimensions** $\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum(\vec{r} \times \vec{F}) = 0$ $\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$ $\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$		
4.9: Reactions at Supports and Connections for a Three-Dimensional Structure		
Chapter 5: Distributed Forces: Centroids and Centers of Gravity		
5.9: Forces on Submerged Surfaces $w = bp = b\gamma h$	[areas of geometric shapes: circle, triangle, etc.]	[force]
Chapter 6: Analysis of Structures		
6.1: Introduction	[sigma notation] → **To be taught or reviewed as a special math topic** [four operations] [coordinate system]	[force] [Newton's 3rd Law: Action and Reaction]
<u>Trusses</u> **6.2: Definition of a Truss**		
6.3: Simple Trusses		
6.4: Analysis of Trusses by the Method of Joints		
6.5: Joints under Special Loading Conditions		
6.6: Space Trusses		
6.7: Analysis of Trusses by the Method of Sections		
6.8: Trusses Made of Several Simple Trusses		
<u>Frames and Machines</u> **6.9: Structures Containing Multiforce Members**	[trigonometric functions] [coordinate system] [sigma notation] → **To be taught or reviewed as a special math topic** [four operations]	
6.10: Analysis of a Frame		
6.11: Frames Which Cease to Be Rigid When Detached from Their Supports		
6.12: Machines	[sigma notation] → **To be taught or reviewed as a special math topic** [four operations] [trigonometric functions] → **To be taught or reviewed as a special math topic** [coordinate system]	

Table 2. (Continued).

Engineering Analytic Topics & Formulas **Subject: Statics**	**Math & Science Pre-requisite Topics & Skills** **[Math]**	**[Physics]**
Chapter 7: Forces in Beams and Cables		
Cables **7.7: Cables with Concentrated Loads**	[sigma notation] → **To be taught or reviewed as a special math topic** [trigonometric functions] → **To be taught or reviewed as a special math topic** [four operations] [square root]	[force]
7.8: Cables with Distributed Loads $T\cos\theta = T_O \quad T\sin\theta = W \quad T = \sqrt{T_O^2 + W^2} \quad \tan\theta = \frac{W}{T_O}$		
7.9: Parabolic Cable $y = \frac{wx^2}{2T_O}$		
Chapter 8: Friction		
8.1: Introduction	[four operations] [trigonometric functions] → **To be taught or reviewed as a special math topic** [surface]	[force]
8.2: The Laws of Dry Friction. Coefficients of Friction $F_m = \mu_s N \quad F_k = \mu_k N$		
8.3: Angles of Friction $\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N} \rightarrow \tan\phi_s = \mu_s$ $\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N} \rightarrow \tan\phi_k = \mu_k$		
8.4: Problems Involving Dry Friction		
8.5: Wedges		
8.6: Square-Threaded Screws $Q = P\frac{a}{r} \quad L = nP$		
8.7: Journal Bearings. Axle Friction $M = Rr\sin\phi_k \approx Rr\mu_k \quad r_f = r\sin\phi_k \approx r\mu_k$		
8.9: Wheel Friction. Rolling Resistance $\Pr = Wb$	[four operations]	
THE END		

Table 3. Pre-Requisite Mathematics and Science Topics to Be Reviewed Before Teaching the Pre-Calculus Portions of Statics Topics to 9th Grade Students

Pre-Requisites to be Taught or Reviewed	
[Math]	**[Physics]**
1. [areas of geometric shapes: circle, triangle, etc.] 2. [coordinate system] 3. [cross product] → **To be taught or reviewed as a special math topic** 4. [dot product] → **To be taught or reviewed as a special math topic** 5. [four operations] 6. [geometry: point, axis/line, 3D body] 7. [linear algebra] → **To be taught or reviewed as a special math topic** 8. [measurement: time] 9. [Parallelogram Law for the Addition of Force/Vector Graphics] → **To be taught or reviewed as a special math topic** 10. [percent] 11. [problem-solving] 12. [sigma notation] → **To be taught or reviewed as a special math topic** 13. [square root] 14. [surface] 15. [trigonometric functions] → **To be taught or reviewed as a special math topic** 16. [unit conversion] 17. [vector graphics] → To be taught or reviewed as a special math topic	1. [acceleration] 2. [force] 3. [lever] 4. [motion] 5. [Newton's 1st, 2nd and 3rd Laws] 6. [Newton's Law of Gravitation] 7. [scientific inquiry]

Table 4. Pre-Calculus Based Statics Topics That Possibly Could Be Taught at 9th Grade (Chapters and sections)

Chapter/Section	Page Numbers	Number of Pages
Chapter 1: Introduction (pp. 1-13 → 13 pages sub-total. 6 sections out of 6)		
1.1: What Is Mechanics?	1-13	13
1.2: Fundamental Concepts and Principles		
1.3: Systems of Units		
1.4: Conversion from One System of Units to Another		
1.5: Method of Problem Solution		
1.6: Numerical Accuracy		
Chapter 2: Statics of Particles (pp. 15-63 → 49 pages sub-total. 15 sections out of 15)		
2.1: Introduction	15-63	49
2.2: Force on a Particle. Resultant of Two Forces		
2.3: Vectors		
2.4: Addition of Vectors		
2.5: Resultant of Several Concurrent Forces		
2.6: Resolution of a Force into Components		
2.7: Rectangular Components of a Force. Unit Vector		
2.8: Addition of Forces by Summing *x* and *y* Components		
2.9: Equilibrium of a Particle		
2.10: Newton's First Law of Motion		
2.11: Problems Involving the Equilibrium of a Particle. Free-Body Diagrams		
2.12: Rectangular Components of a Force in Space		
2.13: Force Defined by Its Magnitude and Two Points on Its Line of Action		
2.14: Addition of Concurrent Forces in Space		
2.15: Equilibrium of a Particle in Space		
Chapter 3: Rigid Bodies - Equivalent Systems of Forces (pp. 74-145 → 72 pages sub-total. 21 sections out of 21)		
3.1: Introduction	74-145	72
3.2: External and Internal Forces		
3.3: Principle of Transmissibility. Equivalent Forces		
3.4: Vector Product of Two Vectors		
3.5: Vector Products Expressed in Terms of Rectangular Components		
3.6: Moment of a Force about a Point		
3.7: Varignon's Theorem		
3.8: Rectangular Components of the Moment of a Force		
3.9: Scalar Product of Two Vectors		
3.10: Mixed Triple Product of Three Vectors		
3.11: Moment of a Force about a Given Axis		
3.12: Moment of a Couple		
3.13: Equivalent Couples		
3.14: Addition of Couples		
3.15: Couples Can Be Represented by Vectors		
3.16: Resolution of a Given Force Into a Force at *O* and a Couple		
3.17: Reduction of a System of Forces to One Force and One Couple		
3.18: Equivalent Systems of Forces		
3.19: Equipollent Systems of Vectors		
3.20: Further Reduction of a System of Forces		
3.21: Reduction of a System of Forces to a Wrench		

Table 4. (Continued)

Chapter/Section	Page Numbers	Number of Pages
Chapter 4: Equilibrium of Rigid Bodies (pp. 158-210 → 53 pages sub-total. 9 sections out of 9)		
4.1: Introduction	158-210	53
4.2: Free-Body Diagram		
4.3: Reactions at Supports and Connections for a Two-Dimensional Structure		
4.4: Equilibrium of a Rigid Body in Two Dimensions		
4.5: Statically Indeterminate Reactions. Partial Constraints		
4.6: Equilibrium of a Two-Force Body		
4.7: Equilibrium of a Three-Force Body		
4.8: Equilibrium of a Rigid Body in Three Dimensions		
4.9: Reactions at Supports and Connections for a Three-Dimensional Structure		
Chapter 5: Distributed Forces: Centroids & Centers of Gravity (pp. 219-273 → 55 pages sub-total. 0 sections out of 11)		
Chapter 6: Analysis of Structures (pp. 284-342 → 59 pages sub-total. 12 sections out of 12)		
6.1: Introduction	284-342	59
6.2: Definition of a Truss		
6.3: Simple Trusses		
6.4: Analysis of Trusses by the Method of Joints		
6.5: Joints under Special Loading Conditions		
6.6: Space Trusses		
6.7: Analysis of Trusses by the Method of Sections		
6.8: Trusses Made of Several Simple Trusses		
6.9: Structures Containing Multiforce Members		
6.10: Analysis of a Frame		
6.11: Frames Which Cease to Be Rigid When Detached from Their Supports		
6.12: Machines		
Chapter 7: Forces in Beams and Cables (pp. 353-401 → 49 pages sub-total. 0 sections out of 10)		
Chapter 8: Friction (pp. 411-460 → 50 pages sub-total. 8 sections out of 10)		
8.1: Introduction	411-441	31
8.2: The Laws of Dry Friction. Coefficients of Friction		
8.3: Angles of Friction		
8.4: Problems Involving Dry Friction		
8.5: Wedges		
8.6: Square-Threaded Screws		
8.7: Journal Bearings. Axle Friction		
8.9: Wheel Friction. Rolling Resistance	443-450	8
Chapter 9: Distributed Forces: Moments of Inertia (pp. 471-544 → 74 pages sub-total. 0 sections out of 18)		
Chapter 10: Method of Virtual Work (pp. 557-591→ 35 pages sub-total. 0 sections out of 9)		

Table 4. (Continued)

Statistical Summary	
Total Number of Pages Covered by Text (Excluding "Review and Summary for Chapters," "Review Problems" and "Computer Problems Sections)	509
Total Numbers of Sections Covered Under All Chapters	71 out of 121
Percentage of Pre-Calculus Sections $\%_{\text{Pre-Calculus}} = \left(\frac{\text{Number of Pre-Calculus Sections}}{\text{Total Number of Sections}}\right)(100\%) = \left(\frac{71}{121}\right)(100\%) = 58.7\%$	
Total Numbers of Chapters Covered	6 out of 10
Percentage of Chapters with Pre-Calculus Sections $\%_{\text{Pre-Calculus}} = \left(\frac{\text{Number of Chapters with Pre-Calculus Sections}}{\text{Total Number of Chapters}}\right)(100\%) = \left(\frac{6}{10}\right)(100\%) = 60.0\%$	
Total Number of Pages Covered by Pre-Calculus Portion	285
Percentage of Pre-Calculus Volume $\%_{\text{Pre-Calculus}} = \left(\frac{\text{Number of Pre-Calculus Pages}}{\text{Total Number of Pages}}\right)(100\%) = \left(\frac{285}{509}\right)(100\%) = 56.0\%$	