Joint Beginning and Intermediate Engineering Graphics
8th Week Meeting Lecture Notes
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Topic: Descriptive Geometry Theory

1st Subject: Definition Points, Lines and Planes, Views and Angle between Surfaces

Descriptive geometry is the science of graphical representation and the solution of spatial relationships of points, lines, and planes by means of projections. **Basic definitions:**

- Two points can establish **a line segment**.
- Two line segments having one point in common are **intersecting lines**. Example: line 1-2 and line 2-3 with point 1 (1,1), 2 (2,2), and 3 (3,3).
- Nonintersecting and nonparallel lines are called **skew lines**.
- An **inclined line** appears true length on the plane to which it is parallel. Inclined lines are often classified as frontal (corresponding to front view), horizontal (corresponding to top view) or profile (corresponding to side view).
- The true-length view of an inclined line is always in an inclined position, and the foreshortened views are always in either vertical or horizontal positions.
- If a **direction of sight** is parallel to a true-length view of a line, then that line will appear as **a point** in the resulting view. If a direction of sight is perpendicular to a true-length view of a line, then that line will appear as a **true-length line** in the resulting view. If a direction of sight is neither parallel nor perpendicular to a true-length view of a line, then that line will appear as a foreshortened line in the resulting view.
- **Representation of planes:** Planes can be represented by: (a). intersecting lines, (b). parallel lines, (c). three points not in a straight line, (d). a line and a point not on the line. **Definition of a plane:** a surface on which a straight line joining any two points of the surface lies in the surface. Two straight lines in the same plane must intersect unless the lines are parallel.
- **Piercing Points:** if a straight line is not parallel to a plane, it must intersect that plane in a single point called a piercing point.
- **Intersection of Planes:** the intersection of two planes is a straight line with all points common to two planes.
- **Parallelism:** In any view, parallel lines in space will be projected as parallel lines, or as points when they are perpendicular to the viewing plane, or coincide as a single line when they are parallel to the viewing plane and lined up one behind the other. **Parallel planes:** If a pair of intersecting lines in one plane is parallel to a pair of intersecting lines in a second plane, then the planes are parallel. **Line parallel to plane:** A line is parallel to a plane if it is parallel to a line in the plane; a plane is parallel to a line if it contains a line that is parallel to the given line.
- **Perpendicularity:** A 90° angle appears in true size in any view showing one leg true length, provided that the other leg does not appear as a point in the same view. **Line**

perpendicular to plane: A line perpendicular to a plane is perpendicular to all lines in the plane. For a simpler statement, we can say that: A line perpendicular to a plane is perpendicular to at least two nonparallel lines in the plane. This line will also appear perpendicular and in true length to the edge view of the plane. **Perpendicular planes:** If a line is perpendicular to a given plane, any plane containing that line is perpendicular to the given plane.

2nd Subject: Surfaces and Solids

A. Surfaces

- **Surface:** a 2D geometric entity with an area but without thickness or volume. It may be generated by a generatrix (a straight or curved moving line), and any position of this generatrix is an element of the surface.
- **Circle:** A line tangent to a circle is perpendicular to the radius drawn to the point of tangency, and is in the same plane of the circle in space.
- **Plane:** a "flat", ruled surface generated by a straight line generatrix one point of which moves along another straight line (the directrix), and the generatrix always remain parallel to its original position.
- **Ruled surface:** can be a plane, a single-curved surface, or a warped surface generated by a moving straight line.
- **Single-curved surface:** a developable ruled surface which can be unrolled to coincide with a plane. It is either generated by a straight line generatrix one point of which moves along a plane-curve directrix, and the generatrix always remain parallel to its original position as for a cylinder; or generated by a straight line generatrix one point of which moves along a plane-curve directrix, and the generatrix keeps passing through a fixed point (the vertex) as for a cone. **Cone:** All elements of a cone pass through its vertex. A plane tangent to a cone contains only one straight-line element to its surface, and this straight-line must pass through the vertex of the cone. **Right-circular cones:** All elements of a right-circular cone form the same angle with the base plane of the cone. A plane tangent to a right-circular cone contains one element and forms the same angle as does any element with the base plane of the cone. If two right-circular cones with the same vertex and elements of the same length intersect, the common element or elements of the two cones form the same angles with the two base planes as do the respective sets of elements. **Cylinder:** All elements of a cylinder are parallel to each other and to the axis of the cylinder. A plane tangent to a cylinder contains only one straight-line element of its surface, and is parallel to the remaining elements and to the axis.
- **Double-curved surface of revolution:** a surface generated only by a curved line revolving about a straight line in the plane of the curve (examples: sphere, ellipsoid, and torus). **Sphere:** The surface of a sphere is a double-curved surface that contains no straight-line elements but only infinite number of points equidistant from the center of the sphere. A plane tangent to a sphere contains one and only one point in that surface.
- **Warped surface:** a non-developable ruled surface generated by straight lines moving in various patterns, under the principle that no two adjacent positions of the generatrix

lie in the same plan (examples: conoid, helicoids, hyperboloid, streamlined surfaces of aircrafts and automobiles).

B. Solids

- **Polyhedra:** solids bounded by plane surfaces called **faces** with lines of intersection of faces called **edges**. Convex solids whose faces are all congruent regular polygons are **regular polyhedra**, also called **Platonic solids**, including **tetrahedron** (4 triangles), **cube**, **octohedron** (8 triangles), **dodecahedron** (12 pentagons), and **icosahedron** (20 triangles).
- **Solid of revolution:** a solid generated by revolving a plane figure about an axis in the plane of the figure.
- **Solids bounded by warped surfaces:** these types of solids have no group name. Example: screw thread.

Note: in practice, the term solid refers to any 3D closed form, which may be either solid or hollow.

3rd Subject: Development of Single Solids

A. General concepts: The development of a surface is the unfolded or unrolled surface pattern of a form, which gives the true size of each area of the form that in turn can be folded, rolled, or fabricated into the desired form. The practical application of development includes sheet-metal work (construction of containers, pipes, duct, etc.), stone cutting, pattern making and packaging. **Single-curved surface and surfaces of polyhedra** can be developed. **Warped surfaces and double-curved surfaces** can be developed only approximately by dividing them into sections and substituting for each section a developable, plane or single-curved surface. **If the materials used is pliable enough,** then stretching, spinning, stamping, pressing and other methods can be used to turn the flat sheets into desired shape. **Extra material** must be used for laps or seams in sheet-metal layout. **Bend allowance** must be taken into heating and ventilating fabrication, it is customary to draw development layouts with inside surfaces up. Doing so will cause all bend lines and other markings to directly relate to inside measurements which are important for tanks and other sheet-metal projects.

B. Parallel-line development: prisms and cylinders are solids with parallel lateral edges and can be developed into flat patterns in which this parallelism is retained. It involves the use of a stretch-out line with a length equal to the peripheral of a right section (base or top of the prism, or circumference of the cylinder). See handouts for details.

- **Development of right or oblique prism** is made of a congruent connection of all sides in true shapes and sizes. For the development of oblique prisms, the construction of auxiliary views might be needed to obtain the true shapes and sizes of some surfaces.
- **Development of right cylinder with no slanted top** is made of a combination of a circular shape and a rectangle with the height of the cylinder and the circumference of the circular shape as its two dimensions.
- **Development of right cylinder with slanted top** can be obtained by dividing the top view (circular shape) into equally spaced elements, these elements are projected vertically down to the front view to generate some points of intersection on the slanted edge line in the front view, these points of intersection are projected

horizontally into the development to intersect with vertical construction lines which project from the stretch-out line (equal in length to the circumference of the circular shape) at equally spaced intervals. The points of intersection are connected into a smooth curved line. The circular base and the elliptical top will then be added with the aid of a compass and an elliptical template.

- **Development of oblique circular cylinders** can be obtained in the similar method as for the right cylinder with slanted top but need construction of auxiliary views. See handouts for details.

C. Radial-line development: pyramids and cones have lateral edges that intersect at a common point called vertex, and in their developments, the lateral edges radiate from this vertex.

- **Development of right and oblique pyramids** can be obtained by finding all sides in true shapes and sizes through revolution, and by connecting them congruently. In sheet-metal, the base is usually not needed, but if desired, it should be attached to a longer edge.
- **Development of right-circular cone** can be obtained by using the division of the base into a large number of equal parts, the slant height, the vertex, the projection of points between top and front views and their conversions to the vertex.
- **Development of elliptical oblique cone** can be made in a similar way for the oblique pyramids.

D. Triangulation is the process of dividing a surface into a number of triangles transferable to a development and is used to develop certain forms that are not pyramids or cones in sheet-metal industry. It is primarily used for the development of transition pieces (the ones that connect two differently shaped and sized or skewed-positioned openings), which in most cases are made of plane or conical surfaces and can be developed by the same methods applied to pyramids and cone, and are extensively used in ventilating, air-conditioning and heating constructions. **A triangle** is said to be **"indestructible"** because if the sides have given lengths, then the triangle can be only one shape.

Study Questions:

Play with two or three sticks while referring to the Lecture Notes, and answer the following questions:

- What is the definition of descriptive geometry?

- How many points are needed to establish a line segment?

- What are two intersecting lines?

- What are the skew lines?

- Under what condition can an inclined line be drawn at true length?

- What does the word “foreshortened” mean?
- Under what conditions can a line appear as a true length line, a foreshortened line, and a point?

- What are the four methods to present a plane? What is the definition of a plane?

- What are the two possible relationships between two straight lines in the same plane?
- What is a piercing point?

- How does the intersection of two planes appear?
- What are the three possible appearances of several parallel lines in space in any view?
- What is the definition of a surface? How is a surface generated?
- What is the definition of a plane? How is a plane generated?
- What is a ruled surface?
- How are single-curved surface and double-curved surface generated? What are the common examples of single-curved surfaces and double-curved surfaces?

- How are warped surfaces formed? Give some common examples of warped surfaces.

- What is a polyhedron? What are the five Platonic Solids?

- Can a solid be hollow? Please explain why.

- How does radial-line development work? Give examples of radial-line development.

- In what sense is a triangle said to be “indestructible”?