Module 1C
Solving for Shortest Distances
Between Points, Lines and Planes

This learning module will explore the skills of solving problems of shortest distances between a point and a line, as well as between two skew lines, using both AutoCAD two-dimensional projection, and AutoCAD three-dimensional modeling and measuring approaches. We will also review some basic math skills (equivalent to what is covered in a typical college-level third semester calculus course), which can be used to solve this type of problems. Since the previous modules (*Module 1A* and *1B*), as well as in the *Section 1* of this module have or will explore in great detail the techniques of using AutoCAD tools in two-dimensional projection, we will focus on the basic strategy of solving this type of descriptive geometry problems in two-dimensional projection (rather than on detailed AutoCAD skills) starting from the *Section 4* of this module.

Section 1
Determining the Shortest Distance between Two Lines

In the design of piping systems, it is sometimes necessary to connect two pipes in the three-dimensional space that happen to be neither parallel nor intersecting, with a juncture pipe with the shortest distance possible, and at the correct location relative to the ends of the two given pipes. In descriptive geometry, we classify this type of problems as the "shortest distance between two lines" (or two central axis lines).

Given: Two adjacent views, both showing the lines as foreshortened

When we talk about the distance between two lines, we mean the shortest distance measured along an imaginary construction line that is perpendicular to both lines. The shortest distance between two lines can be found by drawing the shortest construction line that is perpendicular to both lines, from the point view of one line to the other line graphically. The skill to find the shortest distance between two lines is especially important in mining and civil engineering. Two adjacent views are needed in this type of project.

The key to finding the shortest distance between two lines is to draw an auxiliary view (point end view) of one of the lines (line A-B in *Figure 1C-1A*), which turns the above-mentioned shortest construction line to a plane view, showing the line A-B as a point and the imaginary shortest construction line as a true-length line, and allowing the

measurement of the shortest distance between the point view of the line A-B and the other line (line C-D) by drawing a construction line starting from the point end view of the first line and perpendicular to the other line.

To start, in the *Points, Lines, & Planes in the Three-Dimensional Space.dwg* file (inside the *Given Problems* folder, in the *Student Descriptive Geometry Learning Resource CD*); use the **Zoom** tool to zoom in the area under the tile of *Determining the Shortest Distance between Two Lines* and subtitle of *Given: Two adjacent views, both showing the lines as foreshortened*; and keep the **ORTHO** buttons beneath the **Command Line** turned on, and the **Node**, **Endpoint**, **Perpendicular** and **Intersection** running **Object Snap** in the **Drafting Settings** window turned on.
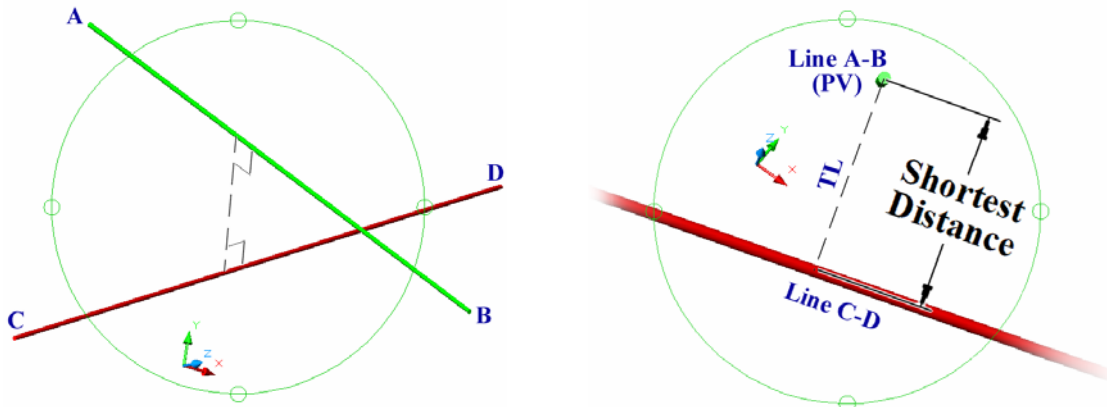

Figure 1C-1A: The imaginary shortest construction line (dashed) perpendicular to both line A-B and line C-D, both shown as foreshortened (left); the line A-B rotated to point view (PV) to shown the true-length view of the shortest distance construction line (right).

In this Section, two adjacent views of line A-B and line C-D in three-dimensional space are given (*Figure 1C-1B*); and in both views, the lines are shown foreshortened. Therefore, a primary auxiliary view is needed to first find the true-length view of line A-B; and then a secondary auxiliary view is projected upon the primary auxiliary view to find the point view (also called the "end view") of the line A-B, and the shortest distance between line A-B and line C-D is measured in the secondary auxiliary view. The step-by-step procedures to accomplish this task are as follows:

First, select one of the given views (the Top view in this example) to draw the Primary Auxiliary view. Use the **UCS** tool with the **3 Point** option to align the **UCS** with one of the foreshortened lines (line A-B in this example); turn **ORTHO** on; use the **Line** tool to draw projection lines from the points A, B, C and D into the Primary Auxiliary view; in addition, create a line that is perpendicular to these projection lines, which

represents the Primary Auxiliary View Ref Plane (*Figure 1C-1C*). The first set of
dimensions for the Primary Auxiliary view is obtained. Return the **UCS** to **Word** settings.
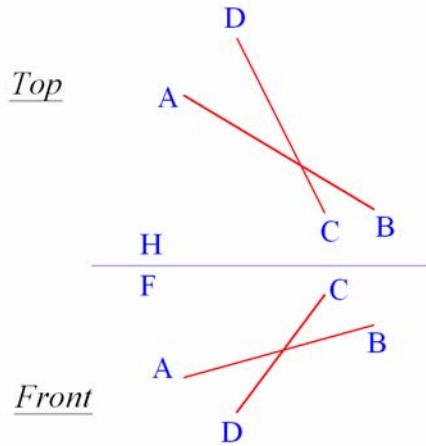

*Figure 1C-1B: The given two
adjacent views showing both line A-B
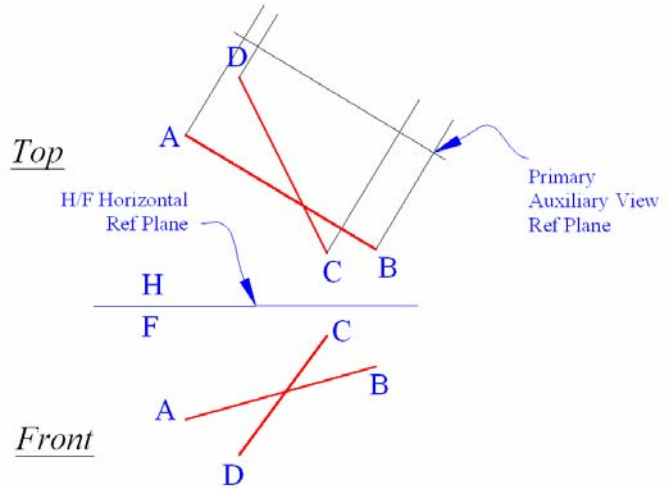and line C-D as foreshortened.*

*Figure 1C-1C: Projecting the Primary Auxiliary view
from the foreshortened line A-B in the Top view.*

Next, obtain the second set of dimensions for the Primary Auxiliary view, in the
view adjacent to the view from where the first set of dimensions is projected (the Front
view in this example, since the Front view is adjacent to the Top view from where the
Primary Auxiliary view is projected). In the Front view, use the **Linear Dimension** tool
to measure the distance from the $\dfrac{H}{F}$ Horizontal Ref Plane to the points C, B, A and D.
Next, in the Primary Auxiliary view, use the **Offset** tool to set off construction lines at
distances C, B, A and D from the Primary Auxiliary View Ref Plane line (alternatively,
use the "construction circle" method). The second set of dimensions for the Primary
Auxiliary view is obtained. Next, cross-referencing among the Primary Auxiliary view,
the Front view and the Top view, determine the points A, B, C and D in the Primary
Auxiliary view; and use the **Line** tool to draw line A-B and line C-D; the line A-B is at
true-length in the Primary Auxiliary view since line A-B in the Top view (the view
adjacent to the Primary Auxiliary view) is parallel to the $H\|1$ reference plane line (*Figure
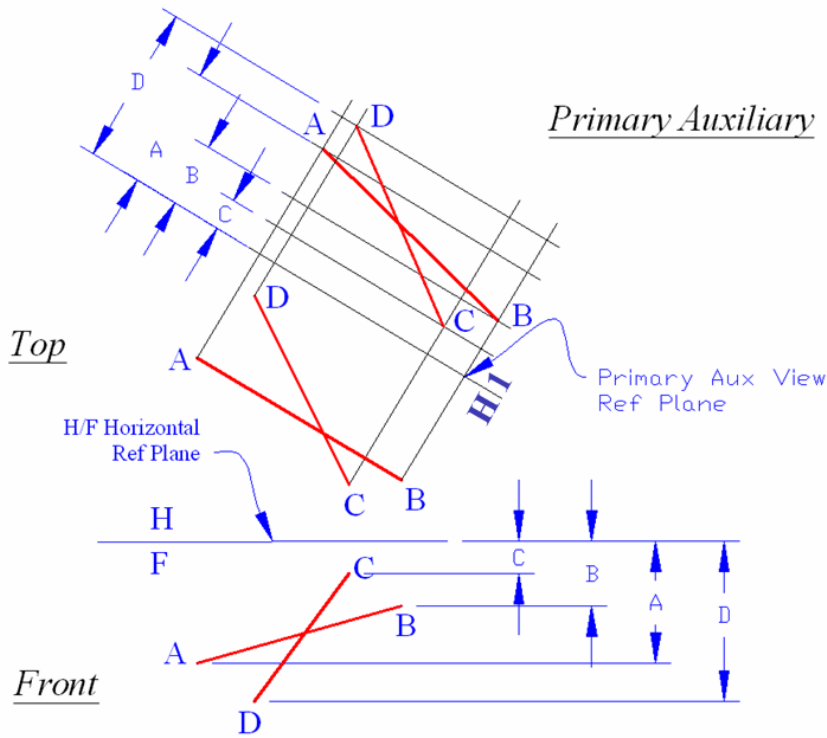1C-1D*).

Figure 1C-1D: Obtaining the second set of dimensions for the Primary Auxiliary view from the Front view.
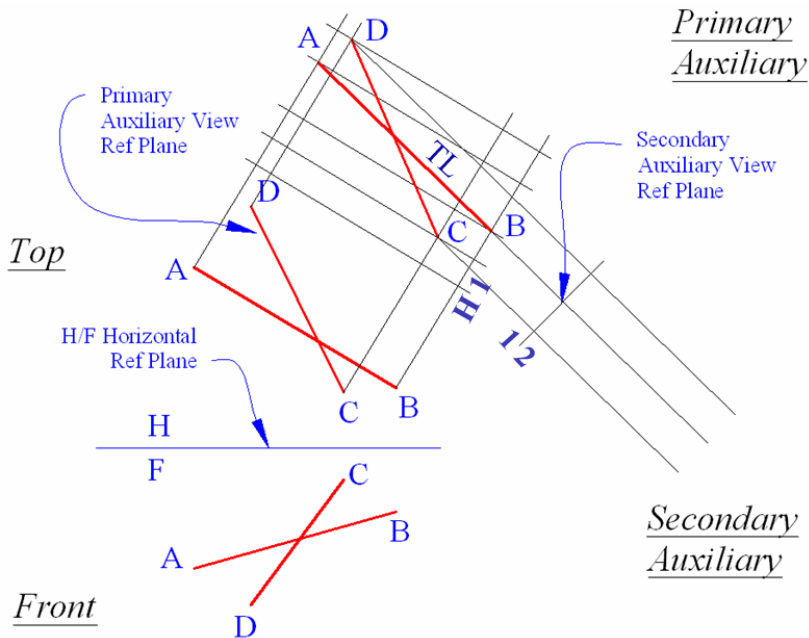

Figure 1C-1E: Projecting the Secondary Auxiliary view from the true-length line A-B of the Primary Auxiliary view; and creating the Ref Plane line.

Next, start to draw the Secondary Auxiliary view. Align the **UCS** with the true-length line A-B in the Primary Auxiliary view; turn the **ORTHO** on; use the **Line** tool to create projection lines from the points C, D and B into the Secondary Auxiliary view; and draw the Secondary Auxiliary View Ref Plane line (1|2) which is perpendicular to these projection lines. The first set of dimensions for the Secondary Auxiliary view is obtained (*Figure 1C-1E*).

Next, go to the Top view (which is adjacent to the Primary Auxiliary view from where the Secondary Auxiliary view is projected), use the **Aligned Dimension** tool to measure the distances from the Primary Auxiliary View Ref Plane line to points C, D and line A-B (C, D and AB as shown in *Figure 1C-1F*).
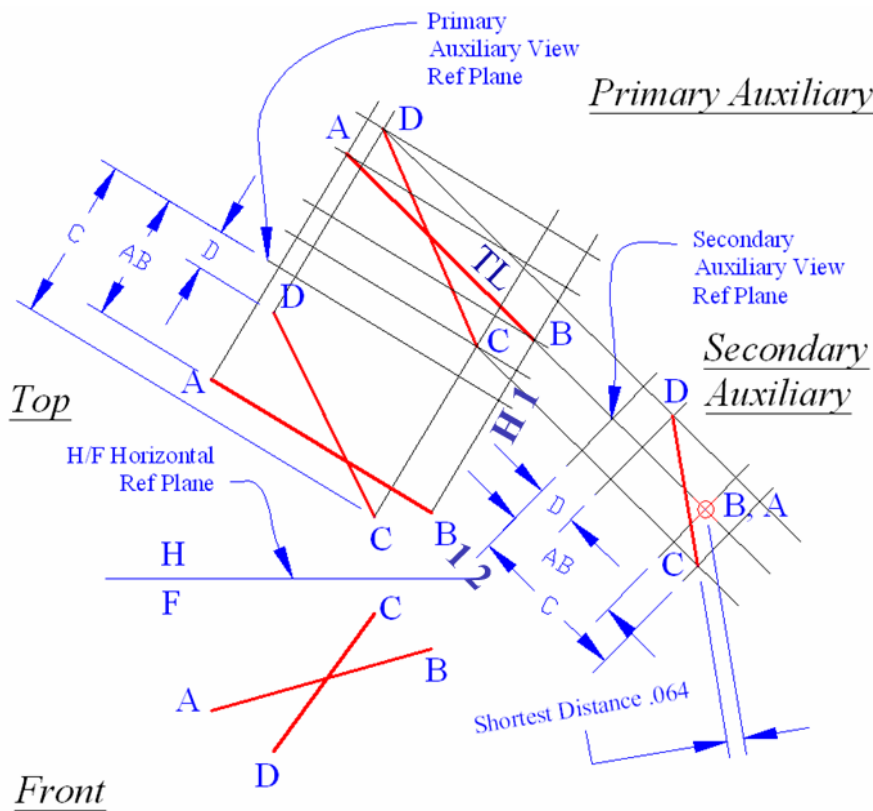

Figure 1C-1F: Completing the Secondary Auxiliary view to turn the line A-B into a point view, and to measure to shortest distance between line A-B and line C-D.

Next, use the **Offset** tool (or use the "construction circle" method) to set off construction lines at the same distances C, D and AB from the Secondary Auxiliary View Ref Plane line (1|2). The second set of dimensions for the Secondary Auxiliary view is

thus obtained. Next, cross-referencing between the Top view and the Primary Auxiliary view, determine the location of points A, B, C and D in the Secondary Auxiliary view. Next, use the **Point** tool to draw point B,A (the point view of the line A-B), and then use the **Line** tool to draw the line C-D, in the Secondary Auxiliary view (*Figure 1C-1F*).

Next, measure the shortest distance between the point B,A (point view of line A-B) and the line C-D, in the Secondary Auxiliary view as shown. There are two ways to accomplish this task:

1. Align the x-axis of the **UCS** with line C-D; then use the **Linear Dimension** tool to pick point C and B,A and obtain the shortest distance as shown in *Figure 1C-1G*;

2. Draw a construction line starting from point B,A and perpendicular to line C-D (with the help of the **Node** and **Perpendicular Object Snap**); and then use the **Aligned Dimension** tool to obtain the shortest distance, as shown in *Figure 1C-1H* and *Figure 1C-1J*. This shortest distance lines has two endpoints, x and y, attached to the line A-B and line C-D somewhere between the endpoints of the lines A-B and C-D, as shown in *Figure 1C-1K*.
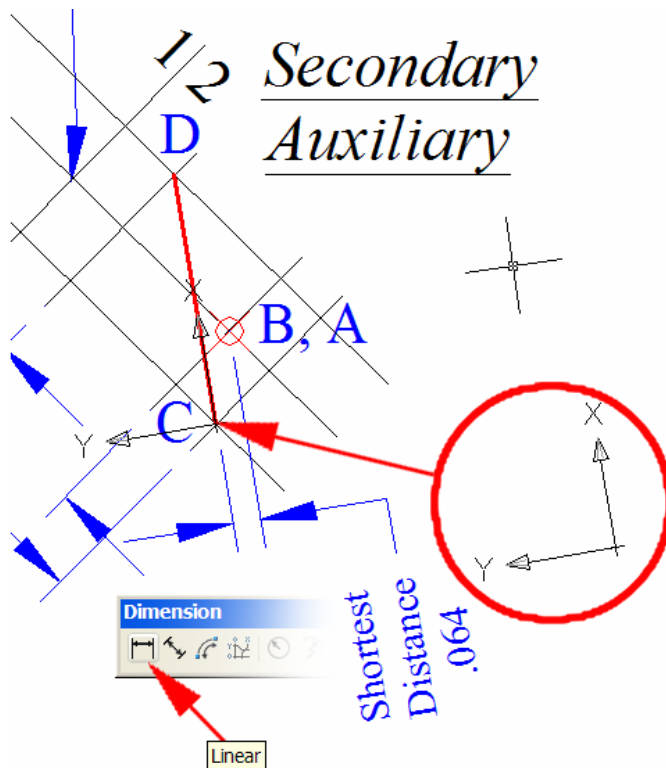

Figure 1C-1G: Aligning the x-axis of the UCS with line C-D; and measuring the shortest distance with the Linear Dimension tool
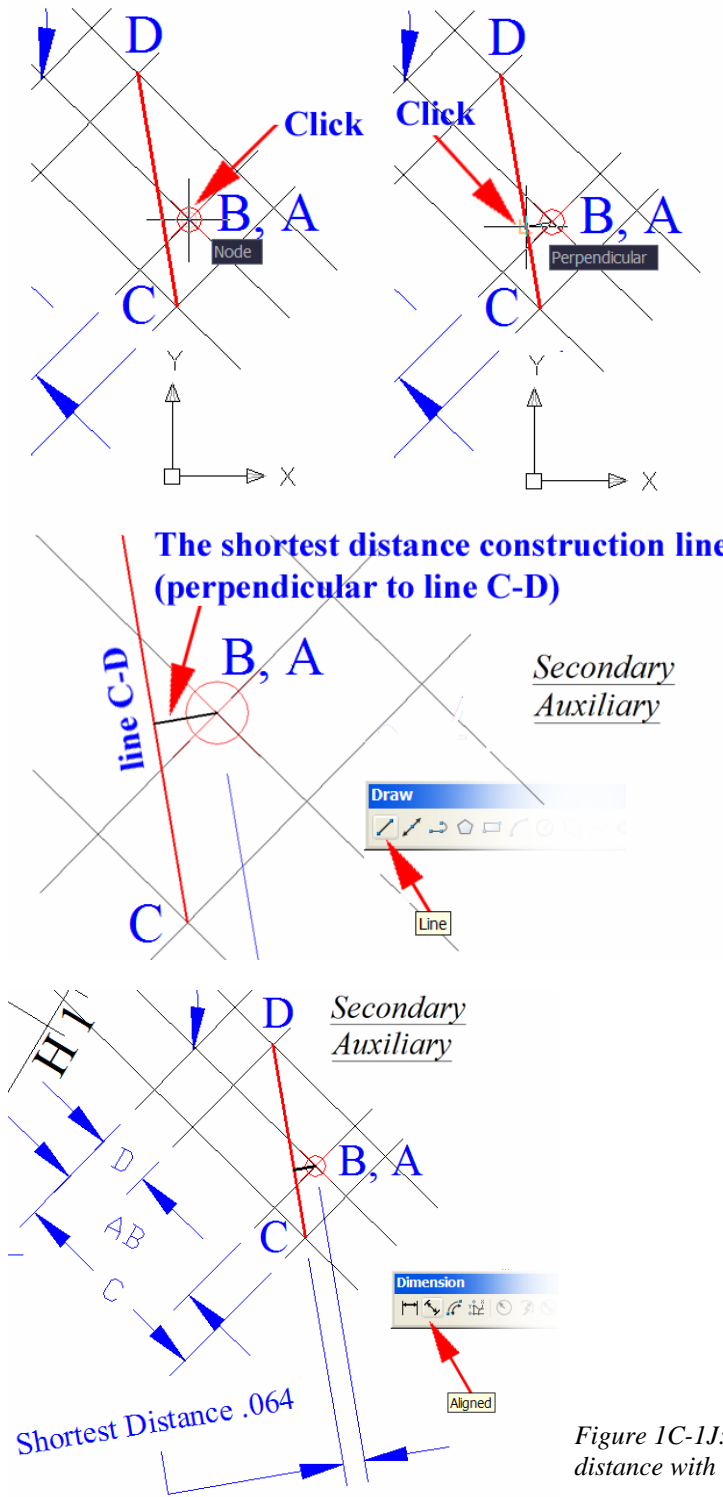
Figure 1C-1H: Creating the shortest distance construction line, using the Line tool with the help of the Node and Perpendicular Object Snap.

Figure 1C-1J: Measuring the shortest distance with the Aligned Dimension tool.
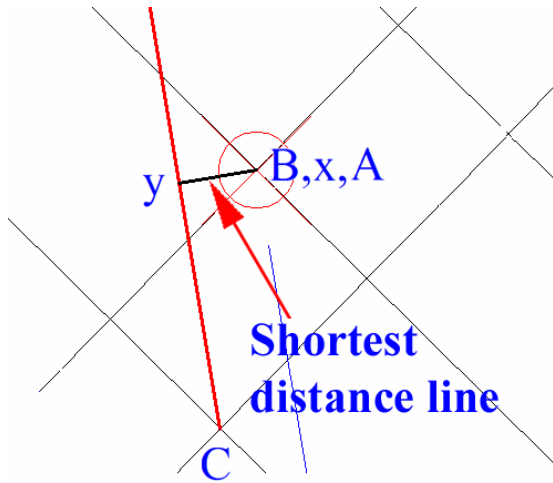
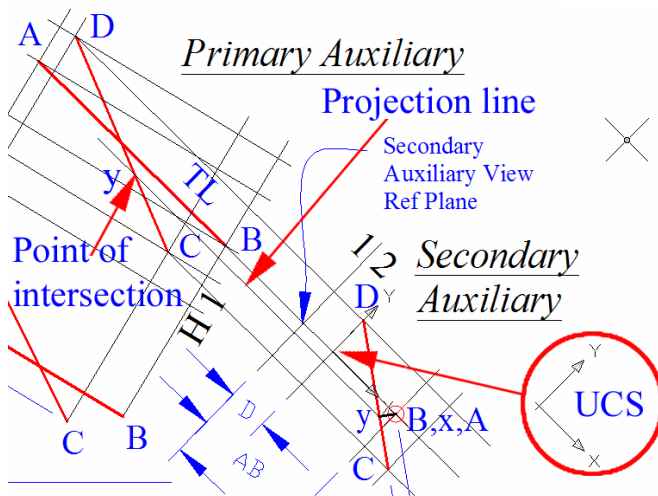*Figure 1C-1K: The endpoints of the shortest line x-y.*

Figure 1C-1L: Projecting the point y from the Secondary Auxiliary view to the Primary Auxiliary view to create the point y on the line C-D.
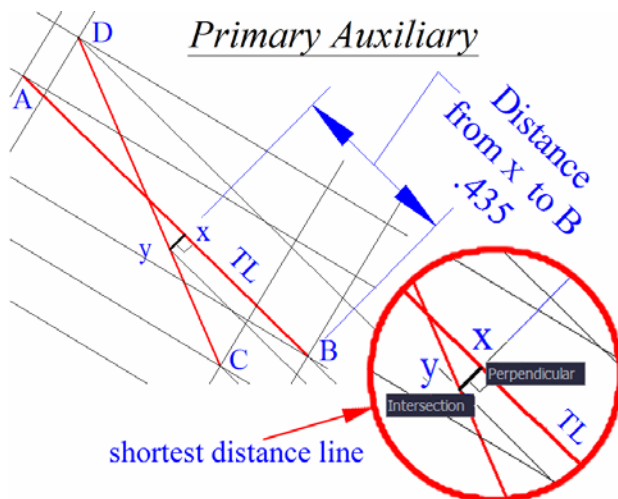

Figure 1C-1M: Creating the foreshortened shortest distance line x-y perpendicular to the true-length line A-B in the Primary Auxiliary view; and measuring the true-length distance x-B.

The locations of the end point x of the shortest distance line relative to the line A-B can be determined on the Primary Auxiliary view where Line A-B is shown at true length. To complete this task, align the x-axis of the **UCS** with the any of the projection lines between the Primary Auxiliary view and the secondary Auxiliary view using the **UCS** tool with the **3 Point** option; turn the **ORTHO** on; use the **Line** tool to create a construction line starting at the point of intersection y on the line C-D in the Secondary Auxiliary view, and ending at the line C-D in the Primary Auxiliary view; this creates the point of intersection y (the point where the shortest distance line is connected to the line C-D, in the Primary Auxiliary view (*Figure 1C-1L*); use the **Line** tool again to create the shortest distance line from point y to point x, with the help of the **Intersection** and **Perpendicular Object Snaps** respectively. Next, use the **Aligned Dimension** tool to measure the distance between the points x and B. This gives the location of the endpoint x of the shortest connecting line with respect to the endpoint B of the line A-B (See *Figure 1C-1M*).

The locations of the end point y of the shortest distance line relative to the line C-D can be determined on an auxiliary view where Line C-D is shown at true length. We can proceed as follows:

First, select one of the given views (the Front view now) to draw the Primary Auxiliary view. Use the **UCS** tool with the **3 Point** option to align the **UCS** with the foreshortened line C-D; turn **ORTHO** on; use the **Line** tool to draw projection lines from the points A, B, C and D into the Primary Auxiliary view; in addition, create a line that is perpendicular to these projection lines, which represents the Primary Auxiliary View Ref Plane. The first set of dimensions for the Primary Auxiliary view is obtained. Return the **UCS** to **Word** settings (*Figure 1C-1N*).
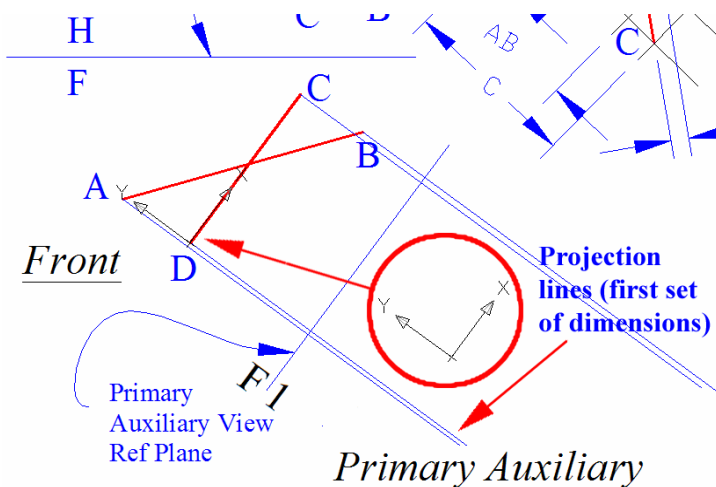

Figure 1C-1N: Projecting the Primary Auxiliary view from the foreshortened line C-D in the Front view, the x-axis of the UCS aligned with line C-D.
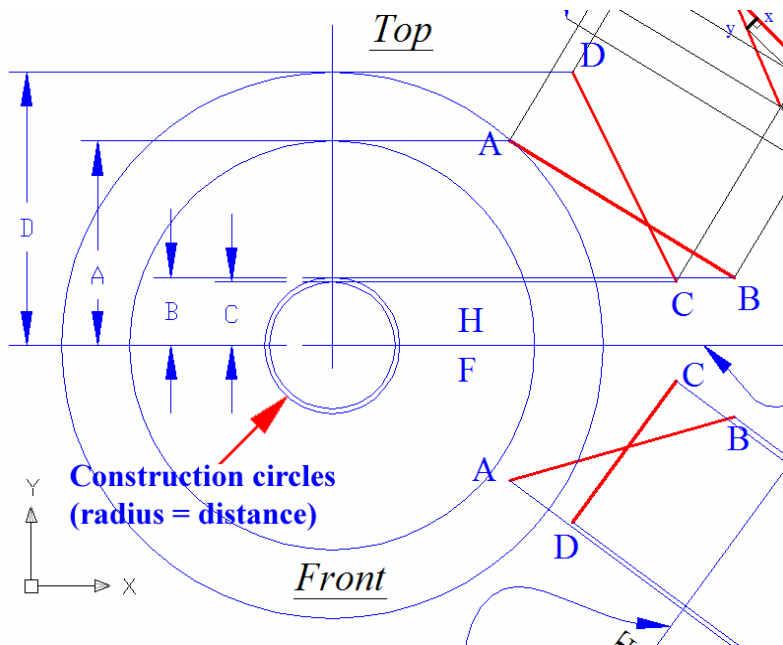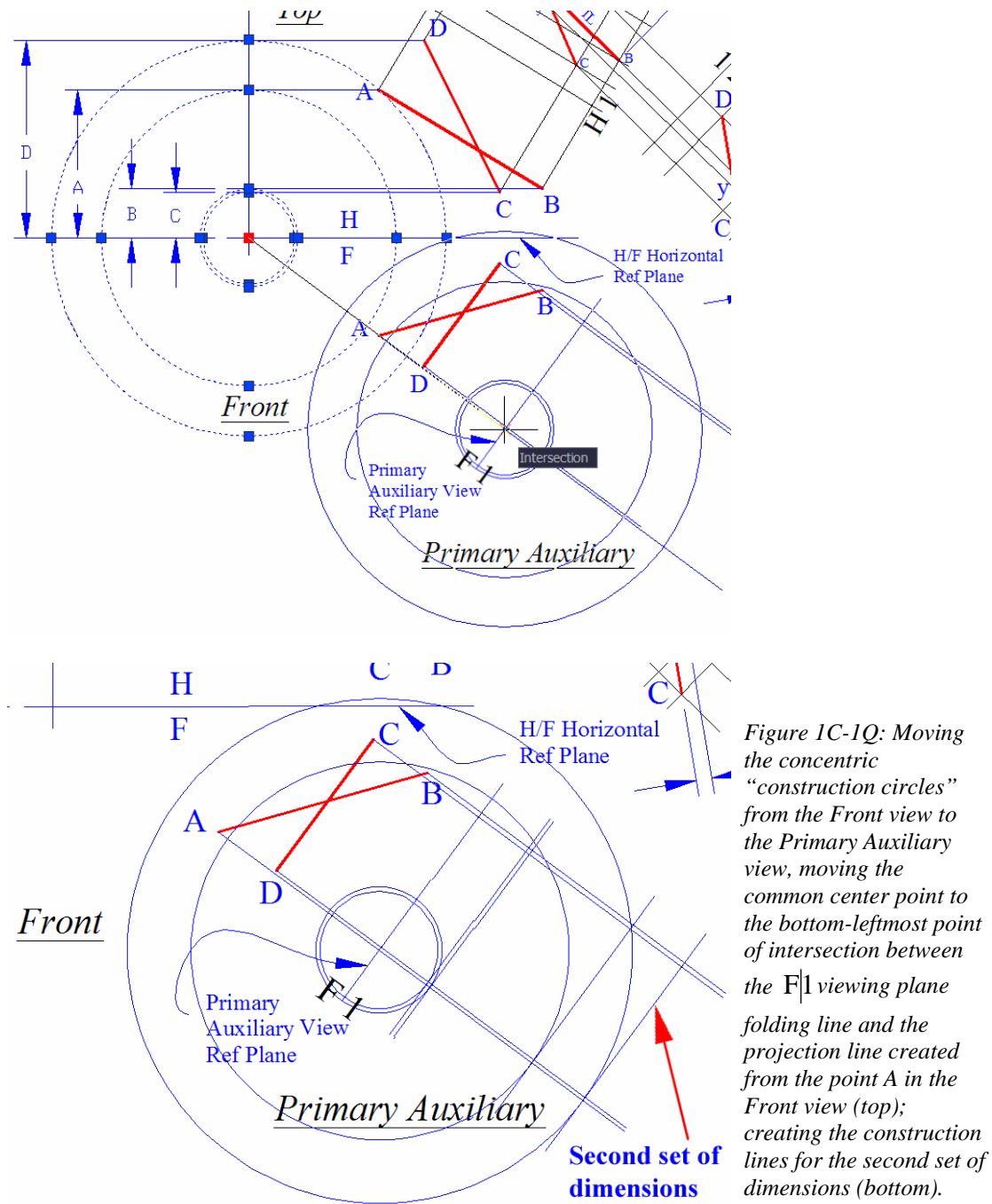
Figure 1C-1P: Obtaining the second set of dimensions for the Primary Auxiliary view from the Top view with the "construction circle" method.

Next, obtain the second set of dimensions for the Primary Auxiliary view, in the view adjacent to the view from where the first set of dimensions is projected (the Top view, which is adjacent to the Front view from where the Primary Auxiliary view is projected), with the "construction circle" method. In the Top view, use the **Line** tool to create horizontal construction lines from the points A, B, C and D as well as a vertical construction line, which gives points of intersection for establishing the radius of the "construction circle;" use the **Circle** tool to create "construction circles" centered at the point of intersection between the vertical construction line and the $\dfrac{H}{F}$ viewing plane folding line (*Figure 1C-1P*). Next, click-select the concentric "construction circles" in the Front view, then click-select the blue-square center grip of the circles to set the center point as the reference starting point for moving; drag the mouse-cursor towards the Primary Auxiliary view, and click once at the bottom-leftmost point of intersection between the $F|1$ viewing plane folding line and the projection line created from the point A in the Front view, with the help of the **Intersection Object Snap** (*Figure 1C-1Q*, left). Use the **Zoom** tools to enlarge the display of the area around the required point of intersection, if necessary, so as to ensure the correct selection of the point. Next, in the Primary Auxiliary view, use the **Copy** tool to create construction lines, by duplicating the $F|1$ viewing plane folding line, from the center of the concentric "construction circles" to the points of intersections between these "construction circles" and the projection line created from the point A in the Front view. The second set of dimensions for the Primary Auxiliary view is obtained (*Figure 1C-1Q*, right).

Figure 1C-1Q: Moving the concentric "construction circles" from the Front view to the Primary Auxiliary view, moving the common center point to the bottom-leftmost point of intersection between the $F|1$ viewing plane folding line and the projection line created from the point A in the Front view (top); creating the construction lines for the second set of dimensions (bottom).

Next, cross-referencing among the Primary Auxiliary view, the Front view and the Top view, determine the points A, B, C and D in the Primary Auxiliary view; and use the **Line** tool to draw line A-B and line C-D; the line C-D is at true-length in the Primary

Auxiliary view since line C-D in the Front view (the view adjacent to the Primary Auxiliary view) is parallel to the F|1 reference plane line (*Figure 1C-1R*).
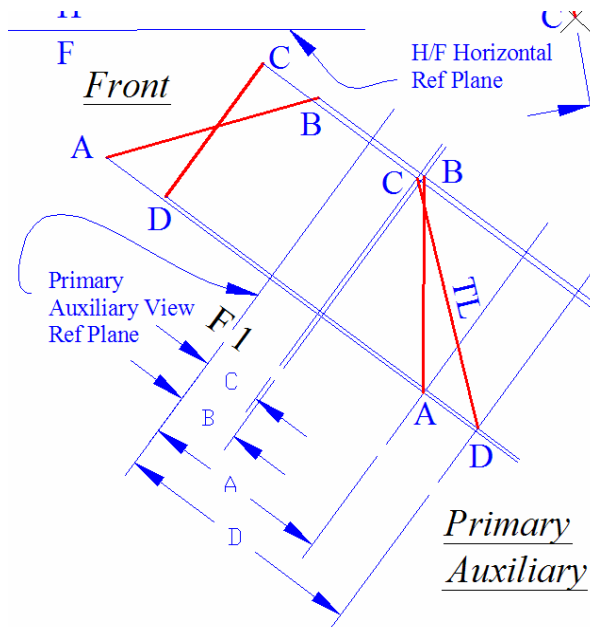

Figure 1C-1R: Completing the Primary Auxiliary view for the true-length line C-D.
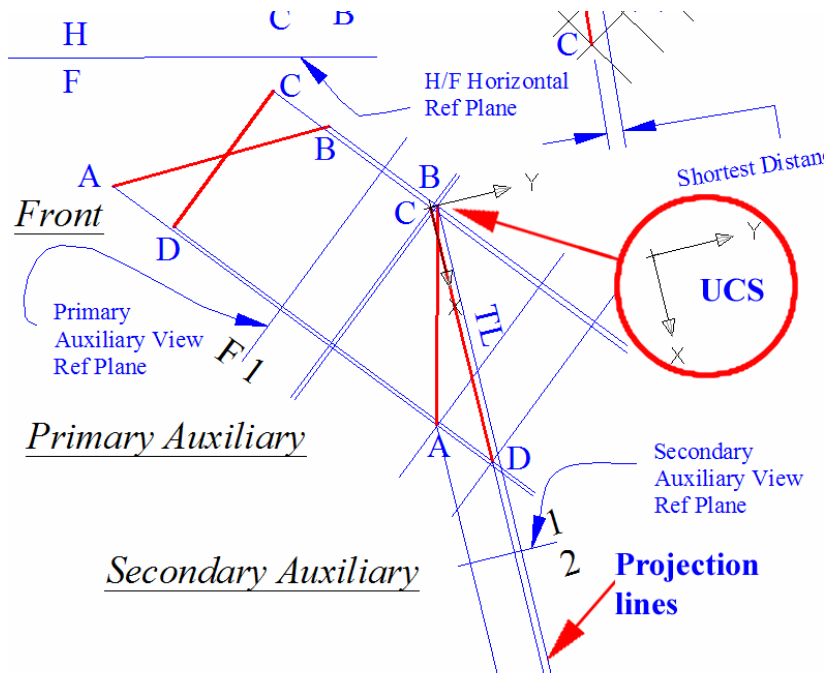

Figure 1C-1S: Projecting the Secondary Auxiliary view from the true-length line C-D of the Primary Auxiliary view; and creating the Ref Plane line.

Next, start to draw the Secondary Auxiliary view. Align the **UCS** with the true-length line C-D in the Primary Auxiliary view; turn the **ORTHO** on; use the **Line** tool to create projection lines from the points A, B and C into the Secondary Auxiliary view; and draw the Secondary Auxiliary View Ref Plane line $\frac{1}{2}$ which is perpendicular to these projection lines. The first set of dimensions for the Secondary Auxiliary view is obtained (*Figure 1C-1S*).
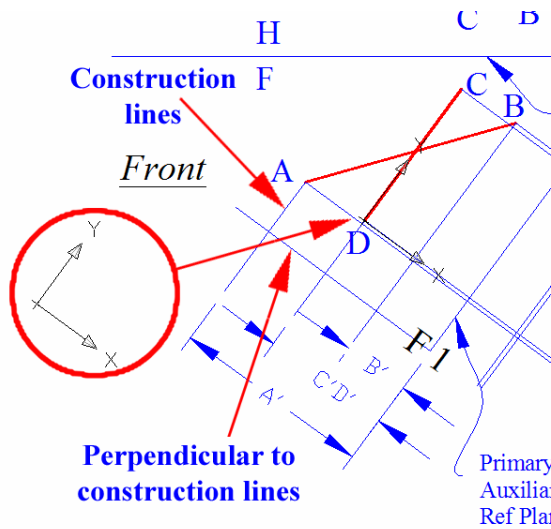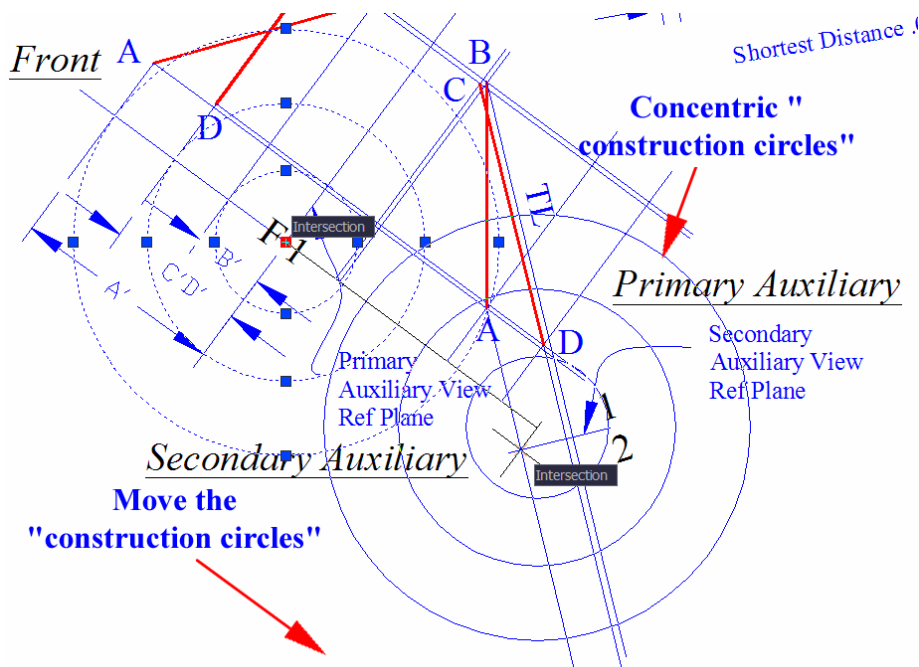

Figure 1C-1T: Obtaining the second set of dimensions for the Secondary Auxiliary view, in the Front view (left); and applying these dimensions with the "construction circle" method to the Secondary Auxiliary view (bottom).

Next, obtain the second set of dimensions for the Secondary Auxiliary view with the "construction circle" method. Go to the Front view (which is adjacent to the Primary Auxiliary view from where the Secondary Auxiliary view is projected); use the **UCS** tool with the **3 point** option to align the x-axis of the **UCS** with any of the projection lines created between the Front view and the Primary Auxiliary view; turn the **ORTHO** on; next, use the **Line** tool to create construction lines from the points B, C, and A; in addition, create a construction line perpendicular to these construction lines, starting from the lower endpoint of the $F|1$ viewing plane folding line, which gives points of intersection for establishing the radius of the "construction circle"(*Figure 1C-1T* left); next, use the **Circle** tool to create concentric "construction circles" centered at the point of intersection between the $F|1$ viewing plane folding line and the construction line perpendicular to it. Next, click-select the concentric "construction circles" in the Front view, then click-select the blue-square center grip of the circles to set the center point as the reference starting point for moving; drag the mouse-cursor towards the Secondary Auxiliary view, and click once at the point of intersection between the $\frac{1}{2}$ viewing plane folding line and the projection line created from the point A in the Primary Auxiliary view, with the help of the **Intersection Object Snap**. Next, in the Secondary Auxiliary view, use the **Copy** tool to create construction lines, by duplicating the $\frac{1}{2}$ viewing plane folding line, from the center of the concentric "construction circles" to the points of intersections between these "construction circles" and the projection line created from the point A in the Primary Auxiliary view. The second set of dimensions for the Primary Auxiliary view is obtained (*Figure 1C-1T*, right).

Next, use the **Copy** tool to create construction lines at the same distances B', C'D' and A' from the Secondary Auxiliary View Ref Plane line $\frac{1}{2}$. The second set of dimensions for the Secondary Auxiliary view is thus obtained. Next, cross-referencing between the Top view and the Primary Auxiliary view, determine the location of points A, B, C and D in the Secondary Auxiliary view. Next, use the **Point** tool to draw point D,C (the point view of the line C,D), and then use the **Line** tool to create the line A-B, in the Secondary Auxiliary view (*Figure 1C-1U*, left). Click-select the "construction circles" and press the **Delete** key on the keyboard to delete them.
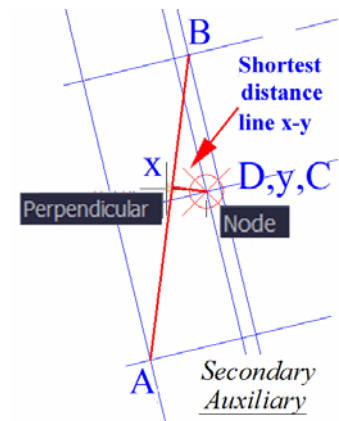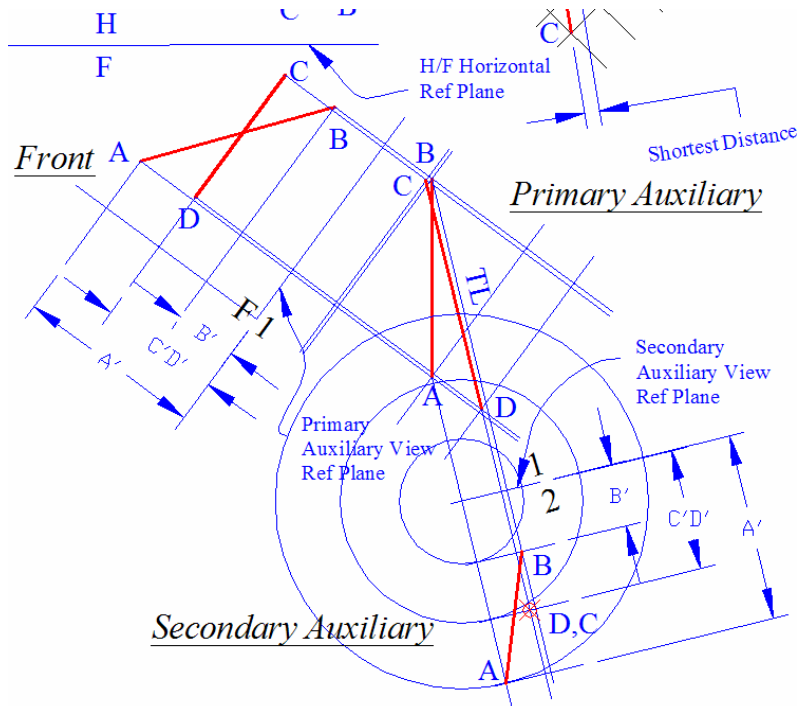
Figure 1C-1U: Completing the Secondary Auxiliary view to turn the line C-D into a point view (left); and creating the shortest distance line, using the Line tool with the help of the Node and Perpendicular Object Snap.
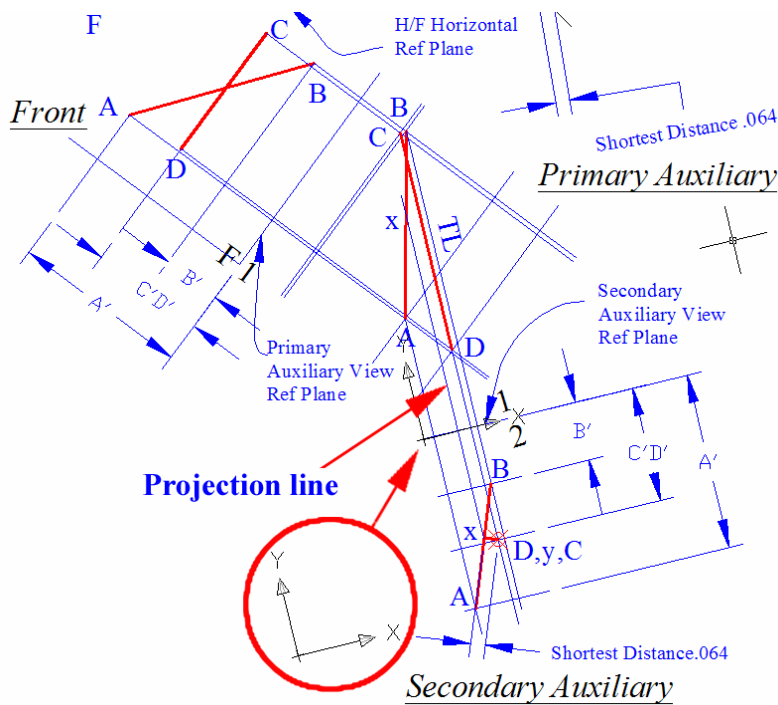
Figure 1C-1V: Measuring the shortest distance between line A-B and line C-D; aligning the x-axis of the UCS with the projection line created between the Primary Auxiliary view and the Secondary Auxiliary view; and projecting the point x of the shortest distance line from the Secondary Auxiliary view to the Primary Auxiliary view.

Next, use the **Line** tool to create the shortest distance line, with the help of the **Node** and **Perpendicular Object Snap**. The endpoint of the shortest distance line connecting to the line A-B is designated as x; and the endpoint of the shortest distance line connecting to the line C-D is designated as y (*Figure 1C-1U*, right), similar to what we have done previously with the Secondary Auxiliary View (where the line A-B is in a point view), which is projected through the Primary Auxiliary view (where the line A-B is shown at true length), from the Top view (where the line A-B is foreshortened), as shown in *Figure 1C-1M*.

Next, use the **Aligned Dimension** tool to measure the shortest distance line x-y, which represents the shortest distance between the point D,C, (point view of line C-D) and the line A-B, in the Secondary Auxiliary view. Notice that the value (0.064) is the same as the one obtained previously, to the thousandths or the third digit after the decimal point. Next, use the **UCS** tool with the **3 Point** option to align the x-axis of the UCS with the projection line created between the Primary Auxiliary view and the Secondary Auxiliary view; turn the **ORTHO** on; and use the **Line** tool to create a projection line starting at the point x of the shortest distance line in the Secondary Auxiliary view (with the help of the **Intersection Object Snap**), and ending at a convenient location past the foreshortened line A-B in the Primary Auxiliary view; this creates the point of intersection x on the foreshortened line A-B in the Primary Auxiliary view (*Figure 1C-1V*).

We mention that the value of the shortest distance (0.064) is the same as the one obtained previously, to the thousandths. How about the precision beyond the thousands? This is a theoretical or philosophical type of question; since for all practical purposes, the thousands is a fairly good benchmark for precision. Nevertheless, we will try to answer this question. Go to the **Dimension→ Dimension Styles…** menu to open the **Dimension Styles Manager Dimension Styles** window; click the **Modify** button to open the **Modify Dimension Style** window; select the **Primary Units** tab, click the ∨ button at the right end of the **Precision** text field and scroll down the menu to select the 0.00000000 (the highest value available); click the **OK** button to return to the **Dimension Styles Manager** window, and click the **Close** button to exit. The values of the shortest distance dimensions change to .06417293 and .06407597 respectively, as shown in *Figure 1C-1W*. This shows that there is a small discrepancy between the two values obtained; and such discrepancy starts at the ten-thousandths or the fourth digit after the decimal point; and this discrepancy is due to complicated reasons including imperfection in graphical projection among the views, especially when the **Dimension/Offset** tool method instead of "construction circle" method is used, which involves the problem of imprecision in the reading of dimension values (as well as the accumulation of such imprecision across several projected views). Other causes might exist; however, they are beyond the scope of this Module; and for all practical purposes, they are insignificant. Now, press the **Ctrl**

key and the **z** key on the keyboard simultaneously to undo the change of dimensional precision setting.
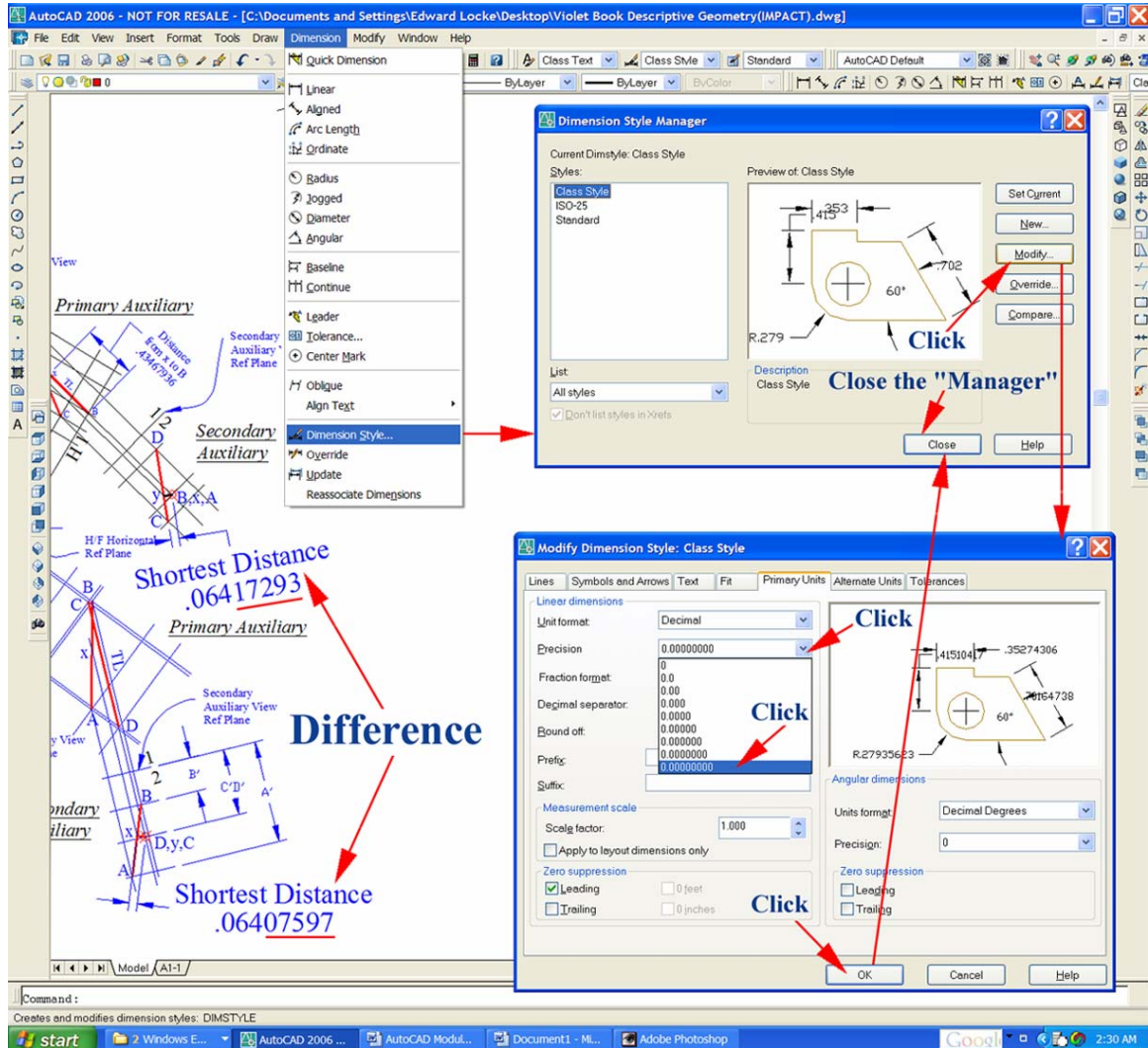

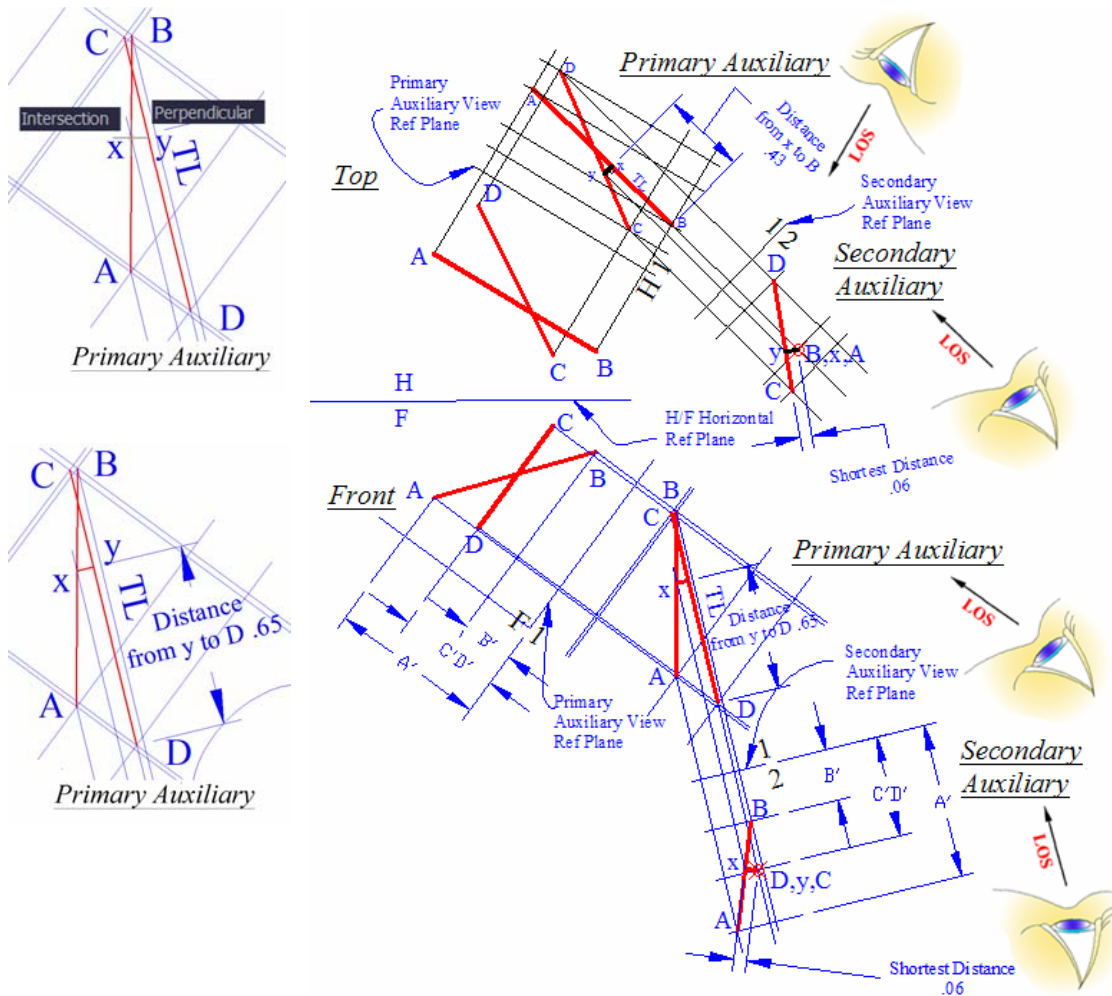*Figure 1C-1W: Changing the precision of dimension figures.*

Figure 1C-1X: Creating the foreshortened shortest distance line x-y perpendicular to the true-length line C-D in the Primary Auxiliary view (top-left). Measuring the true-length distance y-D, with the Aligned Dimension tool (bottom-left). The completed drawing (right).

The locations of the end point y of the shortest distance line relative to the line C-D can now be determined on the Primary Auxiliary view where Line C-D is shown at true length. To complete this task, while the x-axis of the **UCS is still aligned** with the one of the projection lines between the Primary Auxiliary view and the secondary Auxiliary view and the **ORTHO is** on; use the **Line** tool to create the shortest distance construction line starting at the point of intersection x on the line A-B (with the help of the **Intersection Object Snap**), and ending at the point y where the line is perpendicular to the true-length line C-D (with the help of the **Perpendicular Object Snaps**), as shown in *Figure 1C-1X* (top-left). Next, use the **Aligned Dimension** tool to measure the distance between the points y and D. This gives the location of the endpoint y of the

shortest connecting line with respect to the endpoint D of the line C-D, as shown in *Figure 1C-1X* (bottom-left). Return the **UCS** to the default **World** setting. The completed drawing is shown in *Figure 1C-1X* (right). Save the file.

Given: Two adjacent views, one showing one line as true-length

In this part of the Section, two adjacent views of two lines are given, with one line shown at true-length in one of the views (the Front view in this example, as shown in *Figure 1C-1Y*). The line A-B is shown at true-length in the Front view, since in its adjacent view (the Top view), this line is shown parallel to the horizontal $\dfrac{H}{F}$ vowing plane folding line. An auxiliary view is projected from the Front view to find the point view of the line A-B, with the line of sight and the xy-axes of the UCS aligned with the true-length line A-B; and the shortest distance between the line and the point is measured in the auxiliary view, using the same method explored in the first part of this Section.

To start, in the *Points, Lines, & Planes in the Three-Dimensional Space.dwg* file (inside the *Given Problems* folder, in the *Student Descriptive Geometry Learning Resource CD*); use the **Zoom** tool to zoom in the area under the tile of *Determining the Shortest Distance between Two Lines* and subtitle of *Given: Two adjacent views, one showing one line as true-length*; and keep the **ORTHO** buttons beneath the **Command Line** turned on, and the **Node**, **Endpoint**, **Perpendicular** and **Intersection** running **Object Snap** in the **Drafting Settings** window turned on.
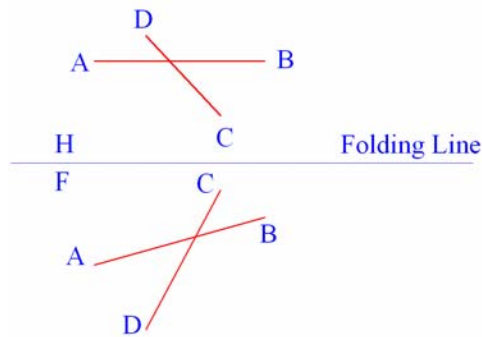

Figure 1C-1Y: The given views.

All steps needed for the solution of the given problem are illustrated in *Figure 1C-1Z.*

The first step is to create the Auxiliary (A-B) view perpendicular to the true-length view of line A-B (in the Front view) so as to obtain the point view of line A-B. Use the **UCS** tool to align the xy-axes of the **UCS** with line A-B in the Front view; turn

the **ORTHO** on; next, use the **Line** tool o draw projection lines from the points A, B, C and D, as well as the Auxiliary View Ref Plane line (labeled as H|1) which are perpendicular to these projection lines; the return **UCS** to **Word** settings.
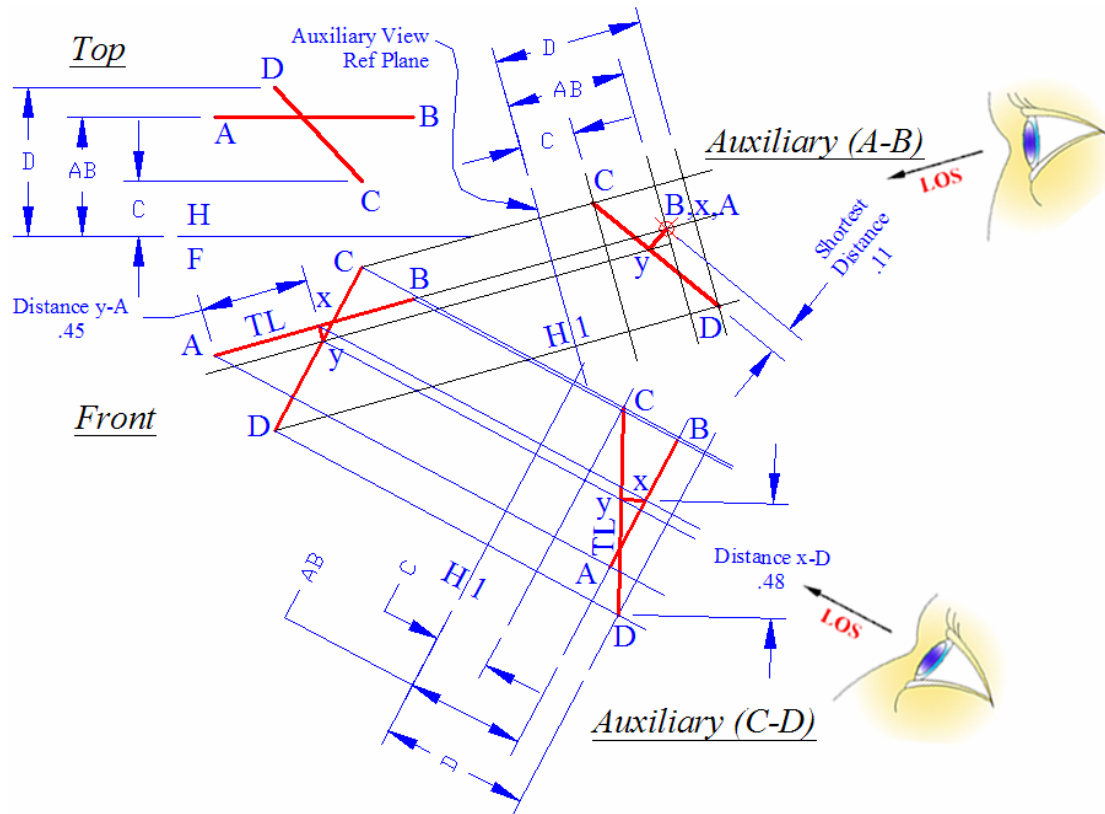

Figure 1C-1Z: The completed Auxiliary views, the shortest distance measured between line A-B and line C-D, and the locational dimensions of the endpoints of the shortest distance line on line A-B and line C-D .

Next, use the **Linear Dimension** tool to measure the distances from the $\dfrac{H}{F}$ Folding Line to point C, D and line A-B in the Top view. Next, go to the Auxiliary view, use the **Offset** tool (or the "construction circle" method) to set off construction lines at same distances from the Auxiliary View Ref Plane line (H|1); next, use the **Point** and **Line** tool to create the point view of the line A-B and the line C-D, and to complete the Auxiliary view. In the Auxiliary view, use the **Line** tool with the help of **Node** and **Perpendicular Object Snaps** to create a construction line from point B,A and perpendicular to line C-D (the "shortest distance" construction line). The ends of this "shortest distance" x and y are attached to the line A-B and to the line C-D respectively.

To locate the endpoint x and y of the shortest distance line in the Front view where the line A-B is shown at true length, use the **Line** tool to create a projection line from the point y in the Auxiliary (A-D) view to the Front view, which gives the point of intersection y on the line C-D in the Front view; from this point of intersection, create a line perpendicular to the true-length line A-B, with the help of the **Intersection** and **Perpendicular** running **Object Snaps**; this gives the point of intersection x on the line A-B; next, use the **Aligned Dimension** tool to measure the distance x-A.

To locate the endpoint y on the line C-D, first create the Auxiliary (C-D) view projected from the foreshortened line C-D in the Front view. Use the **UCS** tool with the **3 Point** option to align the x-axis of the **UCS** with the foreshortened line C-D in the Front view; and with the help of the **ORTHO** as well as the **Intersection** and **Endpoint** running **Object Snaps**, use the **Line** tool to create projection lines from the points A, B, C, D, x and y in the Front view, to the Auxiliary (C-D) view; and create the $H|1$ viewing plane folding line perpendicular to these projection lines. This gives the first set of dimensions for the Auxiliary (C-D) view. To obtain the second set of dimensions for the Auxiliary (C-D) view, go to the Top view which is adjacent t the Front view from where the Auxiliary (C-D) view is projected; use the "construction circle" method to obtain the linear distance from the $\dfrac{H}{F}$ viewing plane folding line to the points A, B, C and D; and transfer these dimensions to the Auxiliary (C-D) view. Use the **Line** tool to create the foreshortened line A-B and the true-length line C-D, as well as the foreshortened shortest distance line x-y (which is perpendicular to the true-length line C-D), in the Auxiliary (C-D) view. Next, use the **Aligned Dimension** tool to measure the distance y-D.

Save and close the file.

Section 2
Solving the Problems of the Shortest Distance between Two Lines in AutoCAD Three-Dimensional Environment

If three-dimensional modeling tools and settings are used to create models of lines, rectilinear pipes or rectilinear cables A-B and C-D in AutoCAD, then the combination of **UCS**, **Line**, **Extension Line**, **Trim** and **Linear Dimension** tools can be used to determine the shortest distance between the two lines A-B and C-D (or between the central axis lines of the rectilinear pipes or cables A-B and C-D). In this part of the Section, we will explore the techniques of creating the three-dimensional models of two pipes A-B and CD, and of determining the shortest distance between them, as well as the

exact location of the endpoints of the shortest distance line relative to the endpoints of the two given lines (or the endpoints of the central axis lines of the two given rectilinear pipes or cables).

In this Section, we will create the three-dimensional models of two pipes in AutoCAD, and use a variety of tools and settings to determine the shortest distance between the central axis of the two pipes, and the exact location where the shortest junction pipe can connect the two pipes, and be perpendicular to both pipes, satisfying the definition of the shortest distance between two lines that we have discussed in Section 16 of this Module.

The given conditions for the two pipes are as follows:

- The coordinates for the central axis line A-B of the Pipe A-B:
  Point A: $X_A = -40$, $Y_A = 90$, $Z_A = -10$; **Absolute Coordinate Entry:** (-40,90,-10).
  Point B: $X_B = -10$, $Y_B = -4$, $Z_B = 160$; **Absolute Coordinate Entry:** (-10,-4,160).
  Note: the unit for all values is [in].
  Length of pipe AB = Distance between endpoints A and B of the central axis line A-B:

$$D_{AB} = \sqrt{(\Delta X_{AB})^2 + (\Delta Y_{AB})^2 + (\Delta Z_{AB})^2} = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (Z_B - Z_A)^2}$$

$$= \sqrt{[(-10)-(-40)]^2 + [(-4)-(90)]^2 + [(160)-(-10)]^2}$$

$$= \sqrt{[-10+40]^2 + [-4-90]^2 + [160+10]^2}$$

$$= \sqrt{[30]^2 + [-94]^2 + [170]^2}$$

$$= \sqrt{900 + 8836 + 28900}$$

$$= \sqrt{38636} \approx 196.5604233 = 196.56\,[\text{in}]$$

- The coordinates for the central axis line C-D of the Pipe C-D:
  Point C: $X_C = -110$, $Y_C = -30$, $Z_C = 110$; **Absolute Coordinate Entry:** (-110,-30,110).
  Point D: $X_D = 60$, $Y_D = 60$, $Z_D = 50$; **Absolute Coordinate Entry:** (60,60,50).
  Length of pipe CD = Distance between endpoints A and B of the central axis line C-D:

$$D_{CD} = \sqrt{(\Delta X_{CD})^2 + (\Delta Y_{CD})^2 + (\Delta Z_{CD})^2} = \sqrt{(X_D - X_C)^2 + (Y_D - Y_C)^2 + (Z_D - Z_C)^2}$$

$$= \sqrt{[(60) - (-110)]^2 + [(60) - (-30)]^2 + [(50) - (110)]^2}$$

$$= \sqrt{[60 + 110]^2 + [60 + 30]^2 + [50 - 110]^2}$$

$$= \sqrt{[170]^2 + [90]^2 + [-60]^2}$$

$$= \sqrt{28900 + 8100 + 3600}$$

$$= \sqrt{40600} \approx 201.4944168 = 201.49\,[in]$$

- The outer diameter of the cross-section of the pipes: 2 [in].

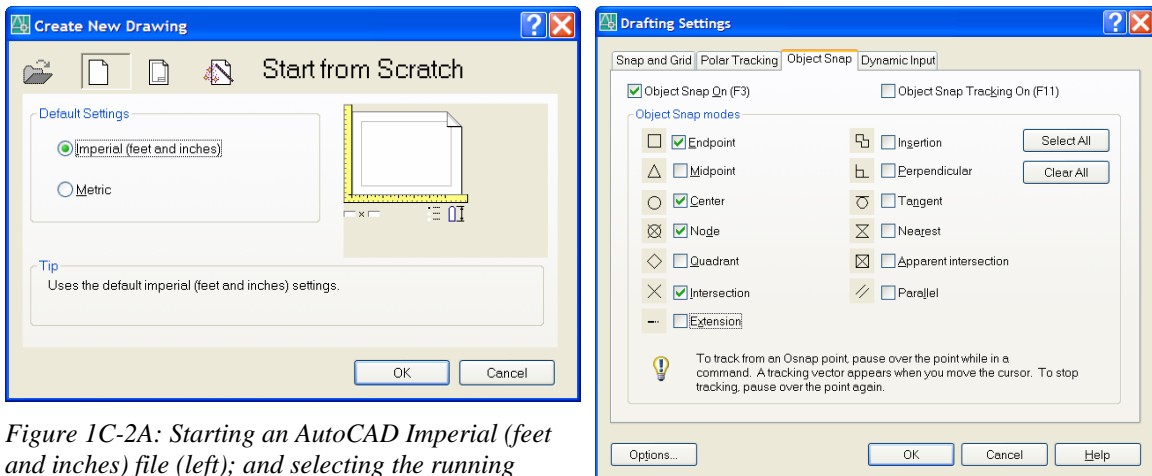- The inner diameter of the cross-section of the pipes: 1.75 [in].

Figure 1C-2A: Starting an AutoCAD Imperial (feet and inches) file (left); and selecting the running Object Snaps (right)

Start a new AutoCAD **Imperial (feet and inches)** file. After the new file opens, type ds in the **Command Line** and press the **Enter** key to open the **Drafting Settings** window; go to the **Object Snap** tab, and check the **Center**, **Intersection**, **Endpoint** and **Node** as running **Object Snaps**, and check the **Object Snap ON (F3)** boxes; click the **OK** button to exit (*Figure 1C-2A*). Go to the **View→3D View→SE Isometric** menu to switch to an isometric view for better visualization in the three-dimensional space (*Figure 1C-2B*). Since we are working in the three-dimensional environment, for better visualization and to ensure accuracy, use the **Zoom Window**, **Zoom Dynamic** (the **Mid-Button** of the three-button mouse), **3D Orbit** and other three-dimensional tools as often as needed. In addition, appropriate **Object Snaps** must be used throughout all operations in the solution of descriptive geometry problems in AutoCAD's three-dimensional space.
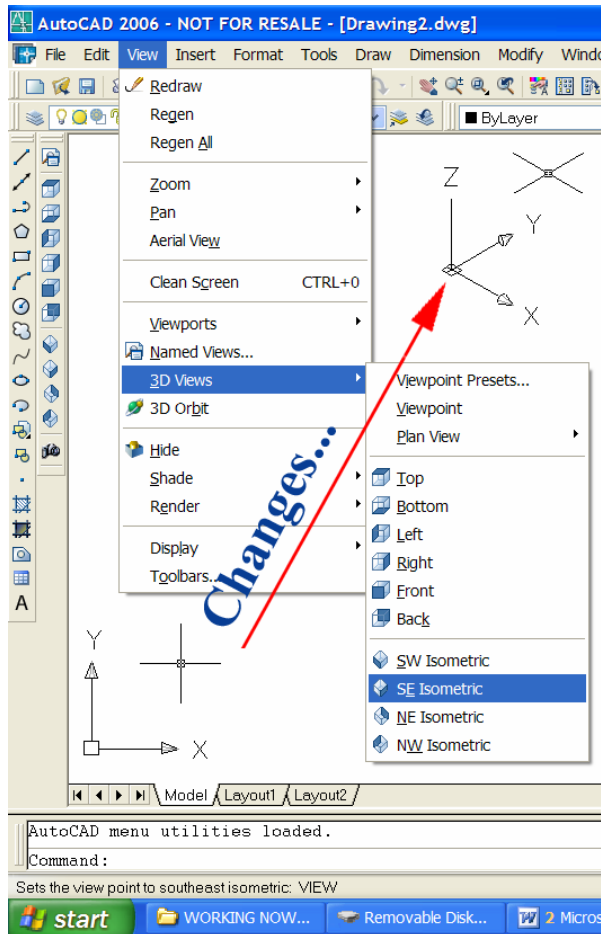
Figure 1C-2B: Switching to an isometric view. The UCS Icon and the Crosshair changed to three-dimensional looks.
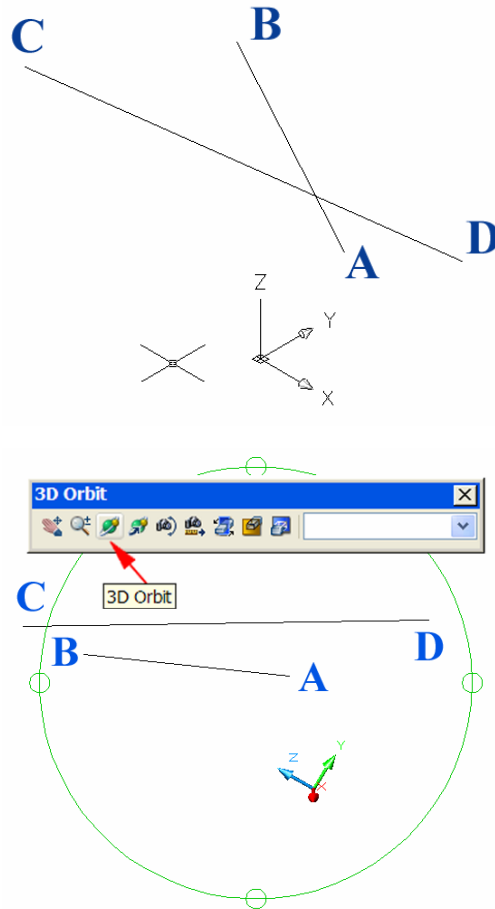
Figure 1C-2C: Creating the central axis lines A-B and C-D for the pipes (top); and rotating the model in 3D space (bottom).

Creating the three-dimensional models of the two pipes

First create the central axes lines A-B and C-D of the pipes AB and CD, with the **Line** tool and the **Absolute Coordinate Entry** method (*Figure 1C-2C*):

Command: l [Type l and press the **Enter** key to invoke the **Line** tool]
LINE Specify first point: -40,90,-10 [Type -40,90,-10 and press the **Enter** key for the **Absolute Coordinate Entry** of the point A]
Specify next point or [Undo]: -10,-4,160 [Type -10,-4,160 and press the **Enter** key for the **Absolute Coordinate Entry** of the point B]
Specify next point or [Undo]: [Press the **Enter** key to invoke the **Line** tool again]
Command: l

`LINE Specify first point:` –110,-30,110 [Type -110,-30,110 and press the **Enter**
key for the **Absolute Coordinate Entry** of the point C]
`Specify next point or [Undo]:` 60,60,50 [Type 60,60,50 and press the **Enter** key
for the **Absolute Coordinate Entry** of the point D. Press the **Enter** key again to exit the
Line tool]

    Next, create the three-dimensional models of the pipes A-B and C-D. Each pipe
needs to be created separately. To create the three-dimensional model of one pipe,
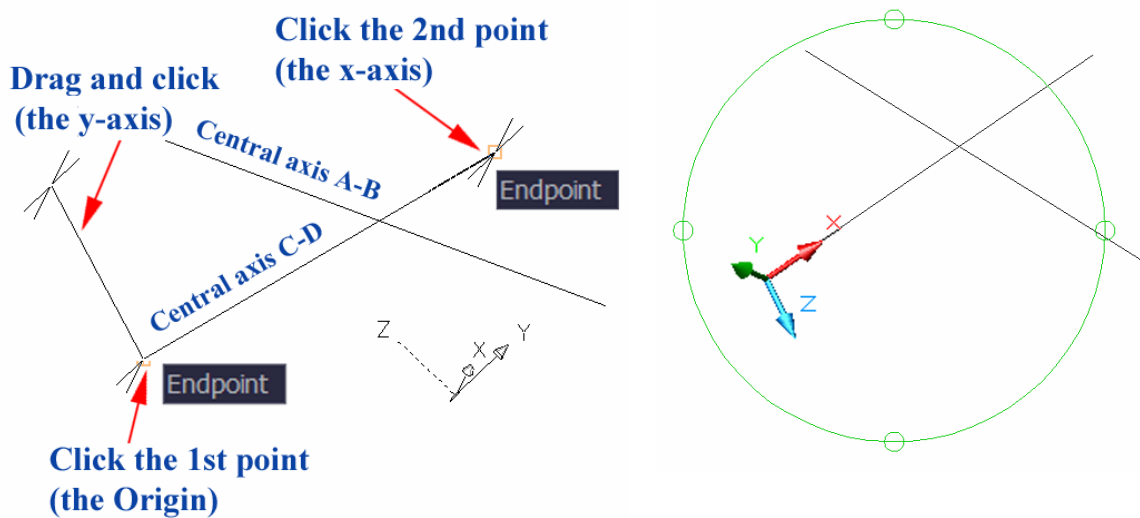proceed as follows:

*Figure 1C-2D: Clicking the first point to set the Origin of the new UCS, the second point to set the x-axis,
and the third point at a convenient location above to set the y-axis (left); the new UCS with the x-axis
aligned with the central axis line C-D (right).*

        Creating the three-dimensional model of the pipe C-D:

    First align the z-axis of the **UCS** with the central axis line C-D of the pipe C-D.
Use the **UCS** tool with the **3 Point** option to align the x-axis with the central axis line C-
D of the pipe C-D, and then use the **UCS** tool with the **y** option to rotate the x-axis by 90°,
such that the z-axis is aligned with the central axis line and the xy-coordinate plane of the
UCS is perpendicular to the central axis of the pipe:

`Command:` ucs [Type ucs and press the **Enter** key to invoke the **UCS** tool]
`Current ucs name:   *WORLD*`
`Enter an option`
`[New/Move/orthoGraphic/Prev/Restore/Save/Del/Apply/?/World]`
`<World>:` 3 [Type 3 and press the **Enter** key to select the **3 Point** option]

Specify new origin point <0,0,0>: [Move the mouse-cursor close to the first point as shown in *Figure 1C-2D* and click once when the **Endpoint** indicator appears]
Specify point on positive portion of X-axis <-109.0000,-30.0000,110.0000>: [Move the mouse-cursor close to the second point as shown in *Figure 1C-2D* and click once when the **Endpoint** indicator appears]
Specify point on positive-Y portion of the UCS XY plane <-110.4679,-29.1162,110.0000>: [Drag the mouse-cursor in the direction above the first point selected and click once]

The x-axis of the **UCS** is aligned with the central axis C-D. In order to create a circular cross-section profile for the pipe C-D, which is perpendicular to the central axis C-D, the **UCS** needs to be rotate around the y-axis by 90°:

Command: ucs [Press the **Enter** key to invoke the **UCS** tool again]
Current ucs name: *NO NAME*
Enter an option
[New/Move/orthoGraphic/Prev/Restore/Save/Del/Apply/?/World]
<World>: y [Type y and press the **Enter** key to select the y-axis as the axis of rotation]
Specify rotation angle about Y axis <90>: [Press the **Enter** key to accept the default 90° angle of rotation]

Next, create the circular cross-section for the pipe with the **Circle** tool:
Command: c [Type c and press the **Enter** key to invoke the **Circle** tool]
CIRCLE Specify center point for circle or [3P/2P/Ttr (tan tan radius)]: [Move the mouse-cursor close to the **Origin** of the **UCS** and click once when the **Endpoint** indicator appears]
Specify radius of circle or [Diameter]: d [Type d and press the **Enter** key to select the **Diameter** option]
Specify diameter of circle: 2 [Type 2 and press the **Enter** key to set the diameter of the cross-section of the pipe C-D as 2 inches]
The cross-section profile of the pipe C-D is created (*Figure 1C-2E*, left).

Next, use the **Extrude** tool to extrude the cross-section profile along the central axis line as the **Path**, and create the pipe.
Command: ext [Type ext and press the **Enter** key to invoke the **Extrude** tool]
EXTRUDE
Current wire frame density: ISOLINES=4
Select objects: [Click-select the circular cross-section profile]
1 found
Select objects: [Press the **Enter** key to end selection]
Specify height of extrusion or [Path]: p [Type p and press the **Enter** key to select the **Path** option]

`Select extrusion path or [Taper angle]:` [Click-select the central axis line C-D as the **Path**]

The three-dimensional model of pipe C-D is created (*Figure 1C-2E*, right).
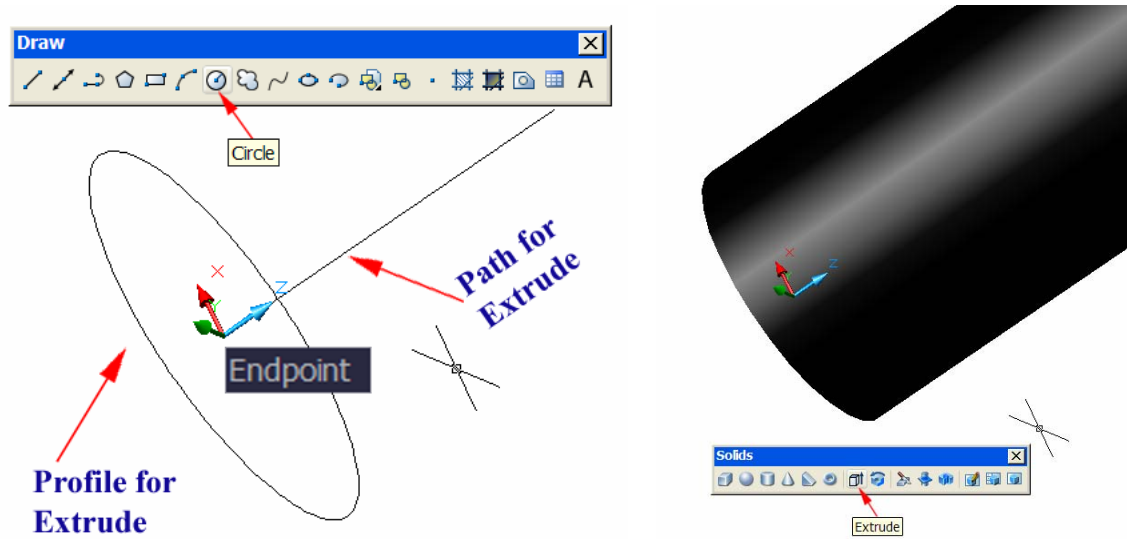

Figure 1C-2E: Creating the circular cross-section profile (left); and extruding the profile along the central axis as the Path to create the pipe C-D (right).

The default color for the pipe is black. Click-select the pipe, go to the **Color Control** drop-down list in the **Property** tool bar, select the **Green** option to change the color of the pipe C-D, for easier identification (*Figure 1C-2F*).

To make the solution of the shortest distance problem simple, we will be happy with a solid pipe for now; and will add the empty space inside the pipe later.

Creating the three-dimensional model of the pipe A-B:

Repeat the same steps to create the pipe A-B. Change the color of the pipe A-B to Magenta. WE are now ready to determine the shortest distance between the pipes A-B and C-D in AutoCAD's three-dimensional space (*Figure 1C-2G*, *Figure 1C-2H* and *Figure 1C-2J*).

Save the file as *Shortest Distance between Two Pipes* in the inside the *Exercise Problems* folder. Save often throughout the project.

Figure 1C-2F: Changing the color of the pipe C-D.
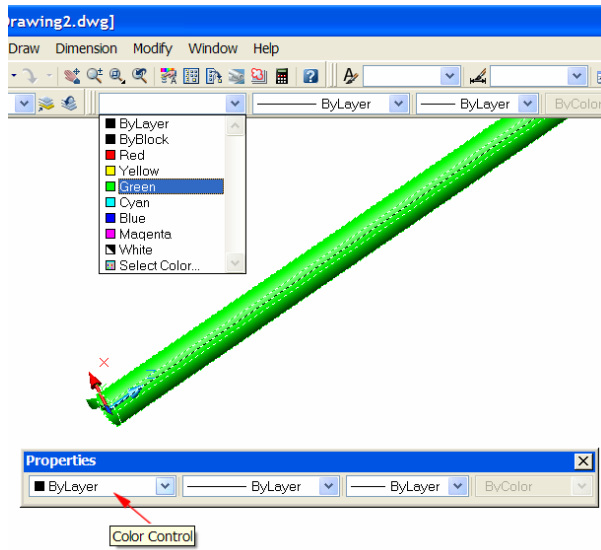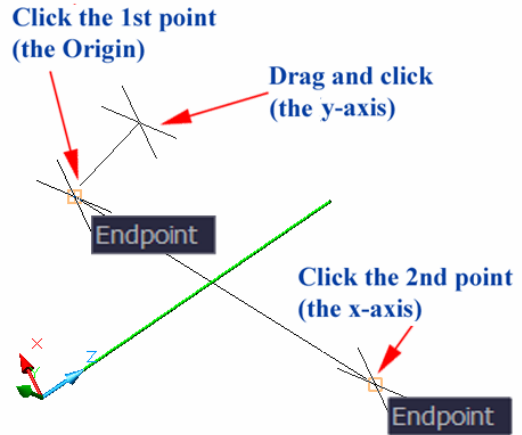

Figure 1C-2G: Setting the new UCS with its x-axis aligned to the central axis of the pipe A-B.
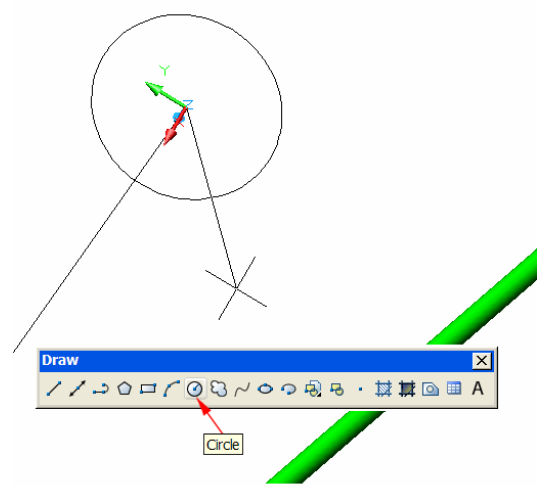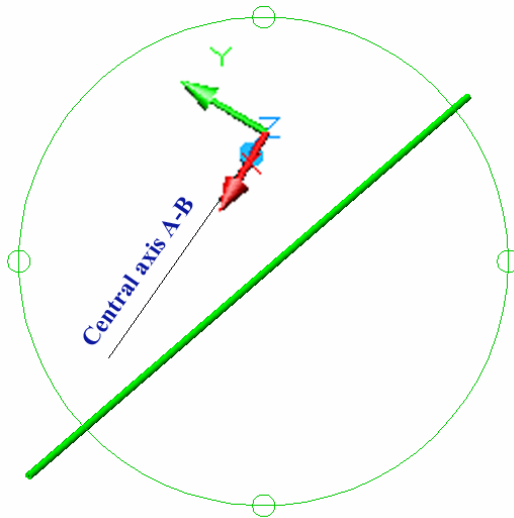

Figure 1C-2H: Aligning the x-axis of the UCS with the central axis A-B and rotating the UCS around its y-axis by 90° to make the xy-coordinate plane perpendicular to the central axis A-B (left); and creating the 2-inch diameter circular cross-section for the pipe A-B (right).
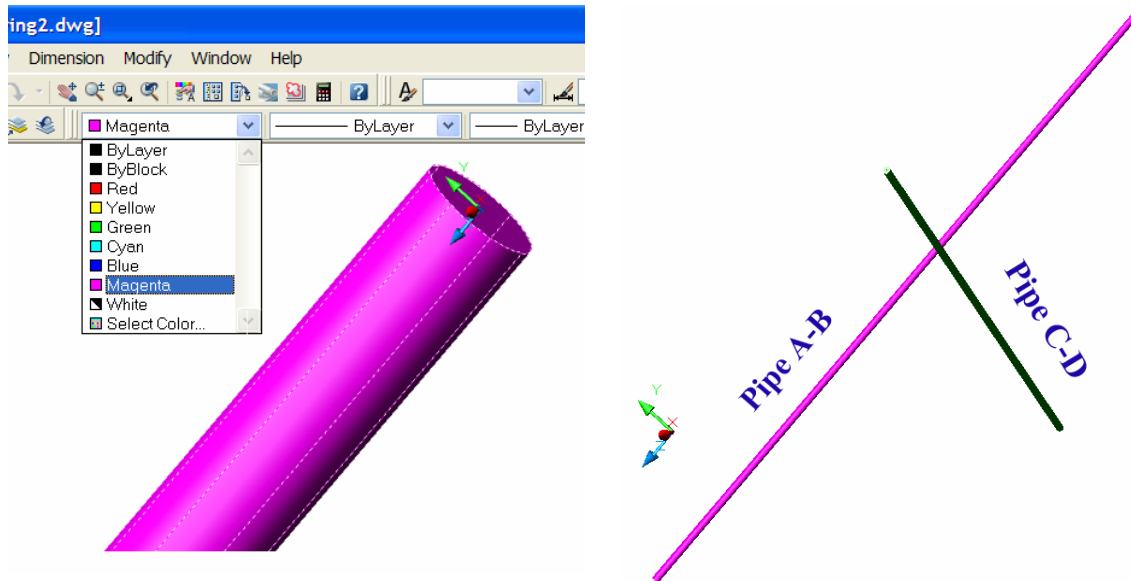
Figure 1C-2J: Extruding the circular cross-section along the Path of central axis A-B into the pipe A-B, and changing its color to magenta (left); the pipes A-B and C-D created in 3D space (right).

Creating the shortest distance line between the central axes of the two pipes with the help of construction lines and the UCS tool

Since the shortest distance line between the two lines in the three-dimensional space is perpendicular to the line of sight of the viewer, and is displayed at true-length only in a view where on the view where one of the lines is displayed as a point (the point view), as we have already discussed in Section 16 of this Module, we should first obtain the point view of the central axis of one of the two pipes; and then create the required construction lines to obtain the shortest distance line. In this problem, the point view of the central axis is equivalent to the plane view of the cross-section of the pipe at one of its ends; and this cross-section is also parallel to the shortest distance line (*Figure 1C-1A*); therefore, we can obtain the true-length view of the shortest distance line be obtaining the plane view of the cross-section of the pipe. The step-by-step procedures to complete this task are described as follows:

First obtain the plane view of the cross-section of one of the two pipes (we select the pink pipe A-B). Notice that right after the creation of the three-dimensional model the pipe A-B, the xy-coordinate plane of the **UCS** is still on the cross at one end of the pipe A-B where the circular profile is created; go to the **View→3D Views→Plane View→Current UCS** menu to obtain the end point view of the central axis of the pipe A-B:
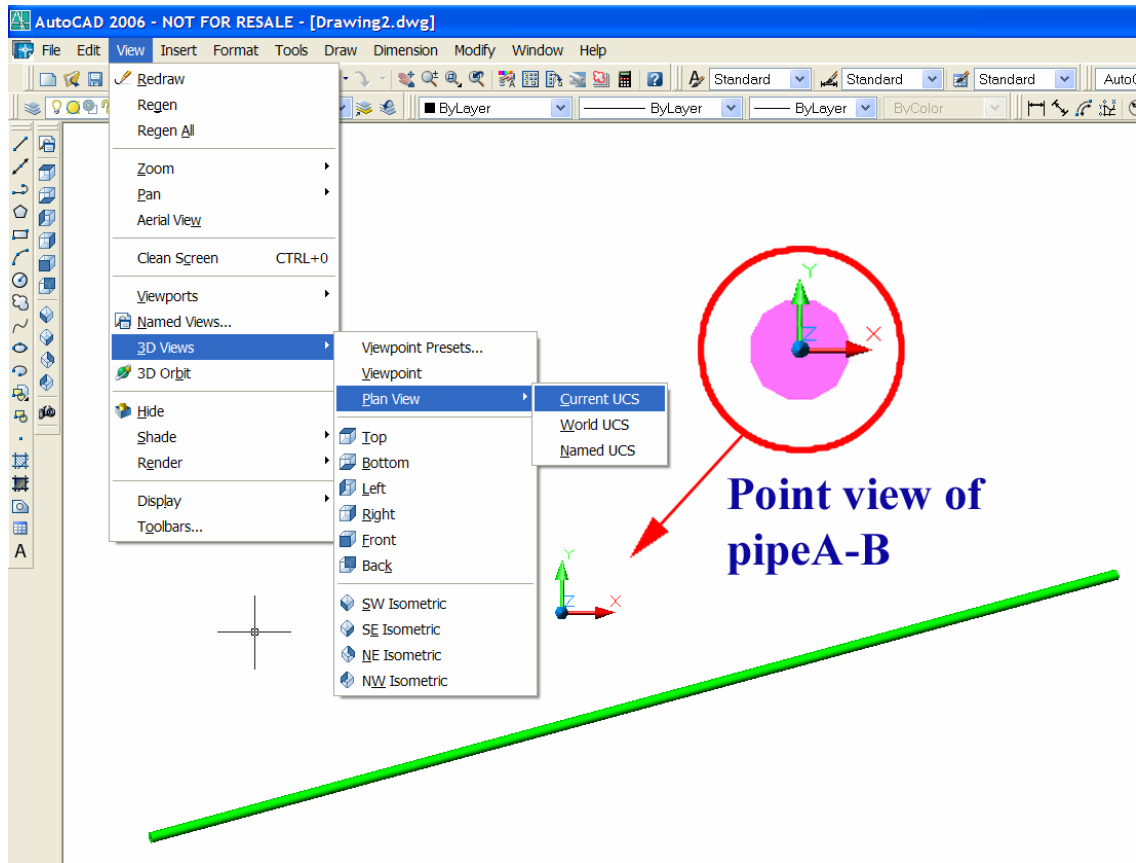
Figure 1C-2K: Obtaining the point view of the pipe A-B (the pink one) on the screen.

Next, use the **Construction Line** tool to create construction line passing through the two center points at both end cross-section of the green pipe C-D, as shown in *Figure 1C-2L*.

Command: xl [Type xl and press the **Enter** key to invoke the **Construction Line** tool]
XLINE Specify a point or [Hor/Ver/Ang/Bisect/Offset]: [Move the mouse-cursor close to one end of the pipe C-D, and click once at the appearance of the **Center Object Snap** indicator]
Specify through point: [Move the mouse-cursor close to one end of the pipe C-D, and click once at the appearance of the **Center Object Snap** indicator]
Specify through point: [Press the **enter** key to exit the **Construction Line** tool]
We use the **Construction Line** tool instead of the **Line** tool, since the **Construction Line** tool creates lines passing through two points and extending to infinity at both ends; this is an important property for solving descriptive geometry problems in three-dimensional space, where the ends of the construction lines needed are sometimes unknown. The **Construction Line** tool is called **Extension Line** in the previous version

of AutoCAD and its **Command Line** shortcut code xl comes from its former name. The **Line** tool, on the other hands, creates line segments with definite length and endpoints, making it difficult to use as a tool for creating construction lines needed for solving descriptive geometry problems in AutoCAD's three-dimensional environment, when the endpoints and length of the construction line are unknown.
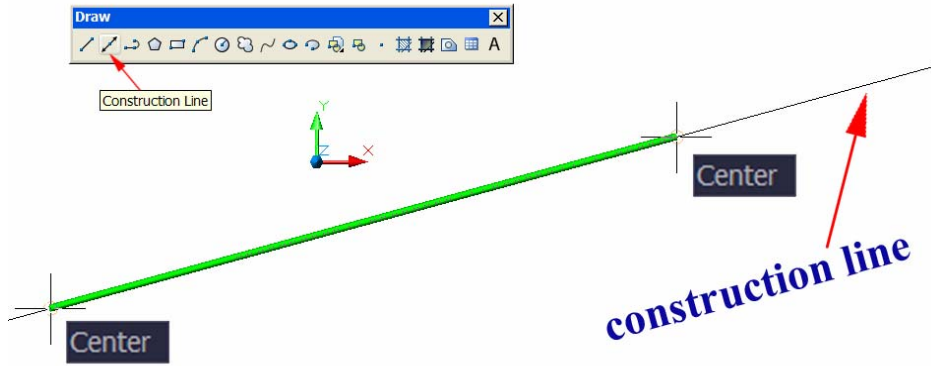
Figure 1C-2L: Creating the construction line passing through the endpoints of the green pipe C-D.
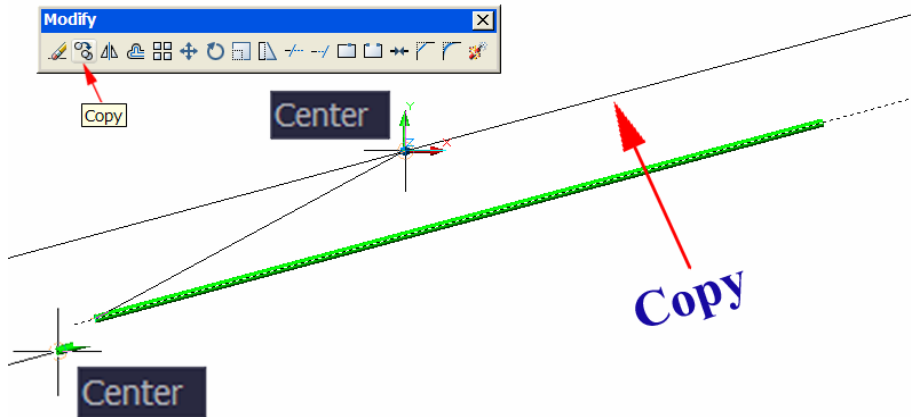
Figure 1C-2M: Copying the construction line.

Next, use the **Copy** tool to create the parallel construction line on the cross-section of the other pipe (the pink pipe A-B with the point view of its central axis shown on the screen), as shown in *Figure 1C-2M*:

Command: co [Type co and press the **Enter** key to invoke the **Copy** tool]
COPY
Select objects: [Click-select the construction line just created] 1 found
Select objects: [Press the **Enter** key to end selection]

New UCS location

Vertical construction line

Figure 1C-2N: Moving the UCS to the center of the end cross-section of the green pipe C-D (top-left).

Creating the vertical construction line with the ORTHO on (top-right).

Creating the new UCS with the x-axis aligned to the central axis of the pipe C-D, and the y-axis on the same plane as the vertical construction line (middle).

The new UCS created (bottom).

The new UCS
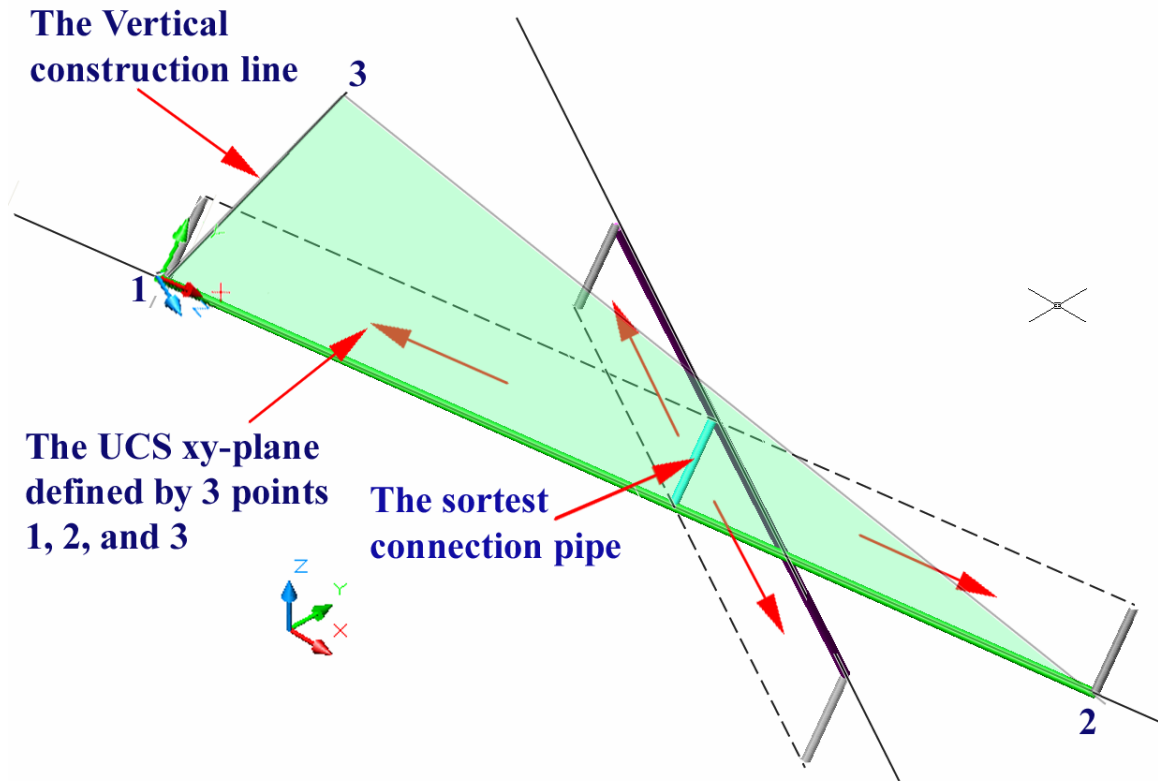
Figure 1C-2N (Continued): The shortest connecting pipe (and the shortest central axis, both corresponding to the shortest line) imagined to be moved from their place of connection to the ends of the pink and green pipes A-B and C-D.
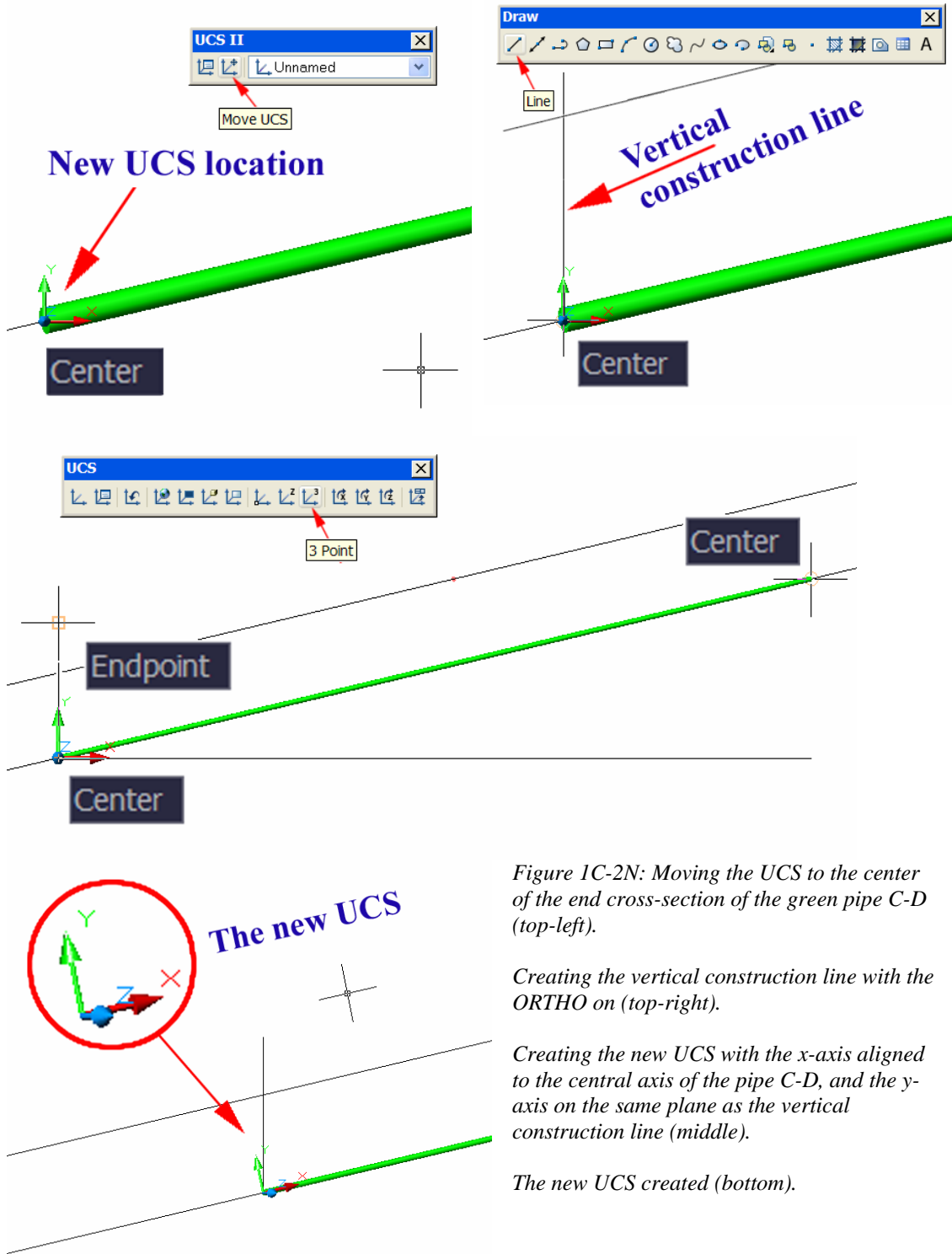
Specify base point or [Displacement] <Displacement>: [Move the mouse-cursor close to one end of the green pipe C-D, and click once at the appearance of the **Center** **Object Snap** indicator]
Specify second point or
<use first point as displacement>: [Move the mouse-cursor close to the center of the cross-section of the pink pipe A-B, which is displayed in plane view; and click once at the appearance of the **Center** **Object Snap** indicator]
Specify second point or [Exit/Undo] <Exit>: [Press the **Enter** key to end selection]

      We now have two parallel construction lines, one passing through the center of the pink pipe A-C; and the other aligned with the central axis C-D of the green pipe. The shortest distance between the two pipes can be measured from a short construction line perpendicular to the two parallel construction lines just created, on the plane view of the cross-section of one pipe (the pink pipe A-B). We can proceed as follows:

Move UCS to the center of one end cross-section of the green pipe C-D (*Figure 1C-2NI*, top-left):

Command: ucs [Type ucs and press the **Enter** key to invoke the **UCS** tool]
Current ucs name: *NO NAME*
Enter an option
[New/Move/orthoGraphic/Prev/Restore/Save/Del/Apply/?/World]
<World>: m [Type m and press the **Enter** key to select the **Move** tool]
Specify new origin point or [Zdepth]<0,0,0>: [Move the mouse-cursor close to one end of the green pipe C-D, and click once at the appearance of the **Center Object Snap** indicator]

Create a vertical line segment starting at the **Origin** of the new **UCS** straight upward, past the construction line, which is copied from the central axis of the green pipe C-D to the center point of the cross-section of the pink pipe A-B, as shown in *Figure 1C-2NI*, top-right:

Command: l LINE [Type l and press the **Enter** key to invoke the **Line** tool]
Specify first point: [Move the mouse-cursor close to the end of the green pipe C-D, and click once at the appearance of the **Center Object Snap** indicator]
Specify next point or [Undo]: [Move the mouse-cursor straight upward past the construction line, with the help of the **ORTHO** and click]
Specify next point or [Undo]: [Press the **Enter** key to exit the **Line** tool]

This vertical construction line, together with the two center point of the end cross-section of the green pipe C-D, constitute the three points needed to establish a **UCS** plane that contains the central axis of the green pipe and is parallel to the end cross-section of the pink pipe A-C. Since the shortest distance between two lines A-B and C-D can display at true-length in the view where one line (central axis A-B of the pink pipe) is shown as a point view, and the shortest distance construction line is contained by a plane that is perpendicular to the line of sight in this particular view, in which all points along the line in point view (central axis A-B of the pink pipe) coincide at a single point, we can think about moving this shortest distance construction line along from its location of connection along both central axes A-B and C-D to the ends of the pipes, as shown in the bottom of *Figure 1C-2N*. We create the vertical construction line as shown in the top-left portion of *Figure 1C-2N* to help creating a **UCS** xy-coordinate plane containing the shortest distance line at one end of the green pipe C-D. We can better understand the process after we follow the instructions, finish the project and use the **3D Orbit** tool to play with the model. Save the file.

Next, Use the **UCS** tool with the **3 Point** option to create the new **UCS** xy-coordinate plane discussed in the end of the previous paragraph. Notice that the central

axes for both pipes have been consumed by AutoCAD during the creation of the three-dimensional models of the pipes; therefore, we will use the center point of the cross-sections of the green pipe C-D as an alternative to the endpoints of the consumed central axis line (*Figure 1C-2N*, middle-left and middle-right).

```
Command: ucs [Type ucs and press the Enter key to invoke the UCS tool]
Current ucs name:  *NO NAME*
Enter an option
[New/Move/orthoGraphic/Prev/Restore/Save/Del/Apply/?/World]
```
<World>: 3 [Type 3 and press the **Enter** key to select the **3 Point** option]
`Specify new origin point <0,0,0>:` [Move the mouse-cursor close to the left end of the green pipe C-D, and click once at the appearance of the **Center** **Object Snap** indicator, for the **Origin** of the new **UCS**]
`Specify point on positive portion of X-axis <1.0000,0.0000,0.0000>:` [Move the mouse-cursor close to the left end of the green pipe C-D, and click once at the appearance of the **Center** **Object Snap** indicator, for the **x-axis** of the new **UCS**]
`Specify point on positive-Y portion of the UCS XY plane`
`<0.0000,1.0000,0.0000>:` [Move the mouse-cursor close to the left end of the green pipe C-D, and click once at the appearance of the **Endpoint** **Object Snap** indicator, for the **y-axis** of the new **UCS**]

Next, start creating the shortest distance line. Use the **Line** tool to create a construction line first, as shown in *Figure 1C-2P* (top-left).

`Command: l` [Type l and press the **Enter** key to invoke the **Line** tool]
`LINE Specify first point:` [Move the mouse-cursor close to one end of the green pipe C-D, and click once at the appearance of the **Endpoint** **Object Snap** indicator]
`Specify next point or [Undo]:  <Ortho on>` [Move the mouse-cursor straight upward along the y-axis of the new **UCS**, with the help of the **ORTHO**, and click once at a convenient point above the construction line on the end cross-section of the pink pipe A-B]
`Specify next point or [Undo]:` [Press the **Enter** key to exit the **Line** tool]

Next, cut off the excessive segment of the shortest distance construction line beyond and above the construction line on the end cross-section of the pink pipe A-B. Use the **Trim** tool with the construction line on the end cross-section of the pink pipe A-B as the **cutting edge**, and the **Project→View** option, as shown in *Figure 1C-2P* (top-right):

```
Command: tr [Type tr and press the Enter key to invoke the Trim tool]
TRIM
Current settings: Projection=View, Edge=None
Select cutting edges ...
```

Select objects or <select all>: [Click-select the construction line on the end cross-section of the pink pipe A-B; it turns into a dotted line]
1 found
Select objects: [Press the **Enter** key to end selection]
Select object to trim or shift-select to extend or
[Fence/Crossing/Project/Edge/eRase/Undo]: p [Type p and press the **Enter** key to select the **Project** option. This option helps to project the geometry from objects that are not created on the xy-coordinate plane of the current **UCS** settings]
Enter a projection option [None/Ucs/View] <View>: v [Type v and press the **Enter** key to select the **View** option. This option helps to project the geometry from objects that are not created on the xy-coordinate plane of the current **UCS** settings, to the xy-coordinate plane of the current **UCS** settings]
Select object to trim or shift-select to extend or
[Fence/Crossing/Project/Edge/eRase/Undo]: [Click-select the shortest distance construction line at the excessive portion. The shortest distance line is completed]
Select object to trim or shift-select to extend or
[Fence/Crossing/Project/Edge/eRase/Undo]: [Press the **Enter** key to exit the **Trim** tool]

An alternative method to cut off the excessive segment of the shortest distance construction line beyond and above the construction line on the end cross-section of the pink pipe A-B is to first create a cutting edge line, which ends at the point of apparent intersection between the shortest distance construction line and the construction line on the end cross-section of the pink pipe A-B, with the help of the **Apparent Intersect Object Snap**, as shown in *Figure 1C-2P* (bottom-left):

Command: l [Type l and press the **Enter** key to invoke the **Line** tool]
LINE Specify first point: [Click any point at a convenient location away from the point of apparent intersection]
Specify next point or [Undo]: app [Type app and press the **Enter** key to invoke the **Apparent Intersect** temporary **Object Snap**]
Of [Move the mouse-cursor close to the point of apparent intersection, and click once at the appearance of the **Apparent Int** **Object Snap** indicator]
Specify next point or [Undo]: [Press the **Enter** key to exit the **Line** tool]

Notice that as shown in *Figure 1C-2P* (bottom-right), the location where the **Apparent Intersect Object Snap** applies depends on the view that is current; if you use the **3D Orbit** tool to rotate the view of the screen, and then try to use the **Line** tool again to pick up this **Object Snap**, then you will find that the location has changed.

After creating the cutting edge line with the **Apparent Intersect** temporary **Object Snap**, the **Trim** tool can be used to cut off the excessive segment of the shortest distance construction line with the default option:

Command: tr [Type tr and press the **Enter** key to invoke the **Trim** tool]
TRIM
View is not plan to UCS. Command results may not be obvious.
Current settings: Projection=UCS, Edge=None
Select cutting edges ...
Select objects or <select all>: [Click-select the cutting edge line; it turns into a dotted line]
1 found
Select objects: [Press the **Enter** key to end selection]
Select object to trim or shift-select to extend or
[Fence/Crossing/Project/Edge/eRase/Undo]: [Click-select the shortest distance construction line at the excessive portion. The shortest distance line is completed]
Select object to trim or shift-select to extend or
[Fence/Crossing/Project/Edge/eRase/Undo]: [Press the **Enter** key to exit the **Trim** tool. Click-select the cutting edge line and press the **Delete** key on the keyboard to delete it]

Measuring the shortest distance between the central axes of the two pipes

We will now use an appropriate **Dimension** tool to measure the length of the shortest distance line. But before we do this, we need to first make some changes to the style of dimensions in AutoCAD, such that the sizes of the dimension figure text and of the arrows, as well as the precision settings will be suitable for this project.

Go to the **Dimension→ Dimension Styles…** menu to open the **Dimension Styles Manager Dimension Styles** window; click the **Modify** button to open the **Modify Dimension Style** window; select the **Primary Units** tab, click the ∨ button at the right end of the **Precision** text field and scroll down the menu to select the 0.000; next, select the **Text** tab, type 5 in the **Text height:** text field to change the height of the dimension figure text to 5 inches for convenient viewing in the three-dimensional environment; next, select the **Lines** tab, type .5 in the **Extend beyond dim lines:** text field, and type .5 in the **Offset from origin:** text field; now, click the **OK** button to return to the **Dimension Styles Manager** window, and click the **Close** button to exit, as shown in *Figure 1C-2Q*.
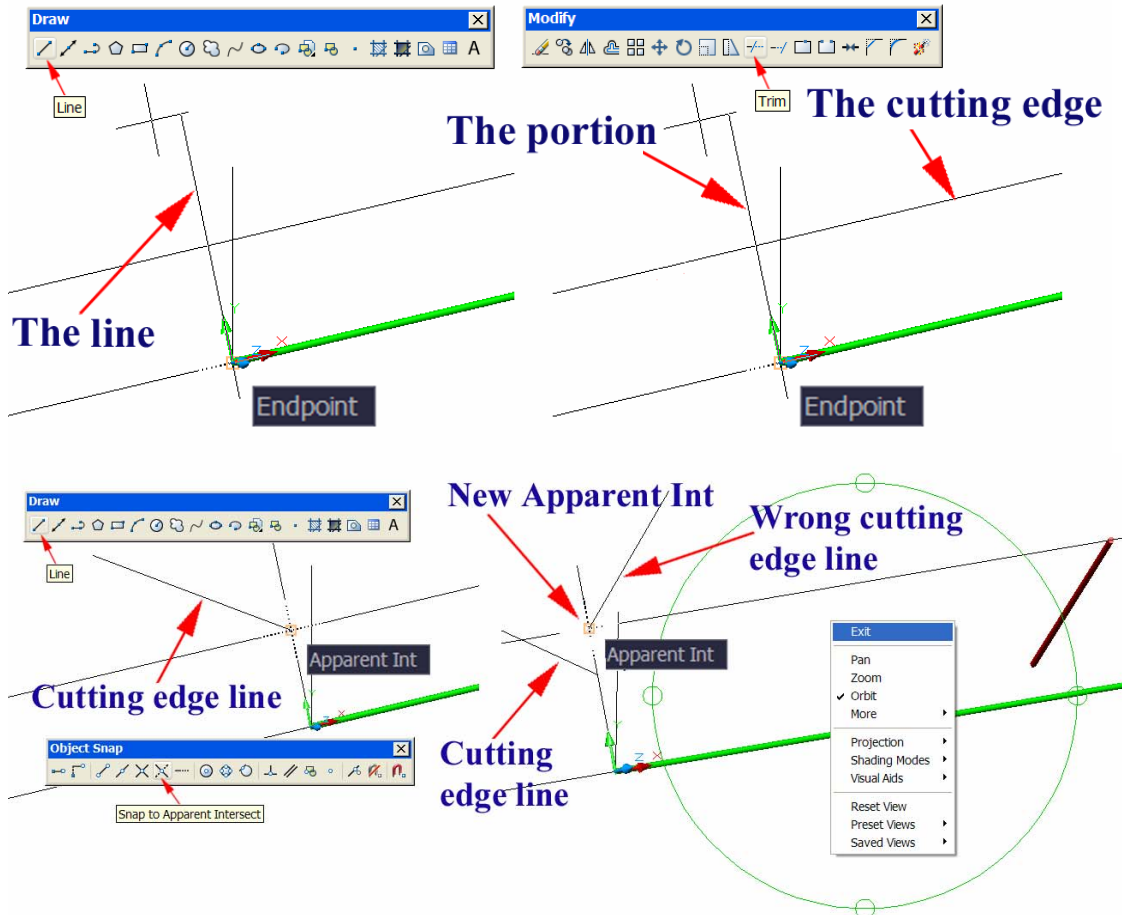
Figure 1C-2P: Creating the shortest distance construction line perpendicular to the two existing parallel construction lines (top-left). Trimming the shortest distance construction line (top-right). Using the Apparent Intersect Object Snap to create a cutting edge line (bottom-left); the dependency of the Apparent Intersect point on the view (bottom-right).

Next, use the **Linear Dimension** or **Aligned Dimension** tool to measure the length of the shortest distance line, as shown in *Figure 1C-2Q* (middle-left). Select either tool from the **Dimensions** tool bar by clicking on the tool icon; right-click to change the tool to a **Quick Dimension** setting; then click-select the shortest distance line; and then drag the mouse-cursor to a convenient location and click one to place the dimension. The dimension figure reads 20.590 (to the third digit after the decimal point, as set up in the **Modify Dimension Style** window, under the **Primary Units** tab and the in the **Precision** drop-down list)
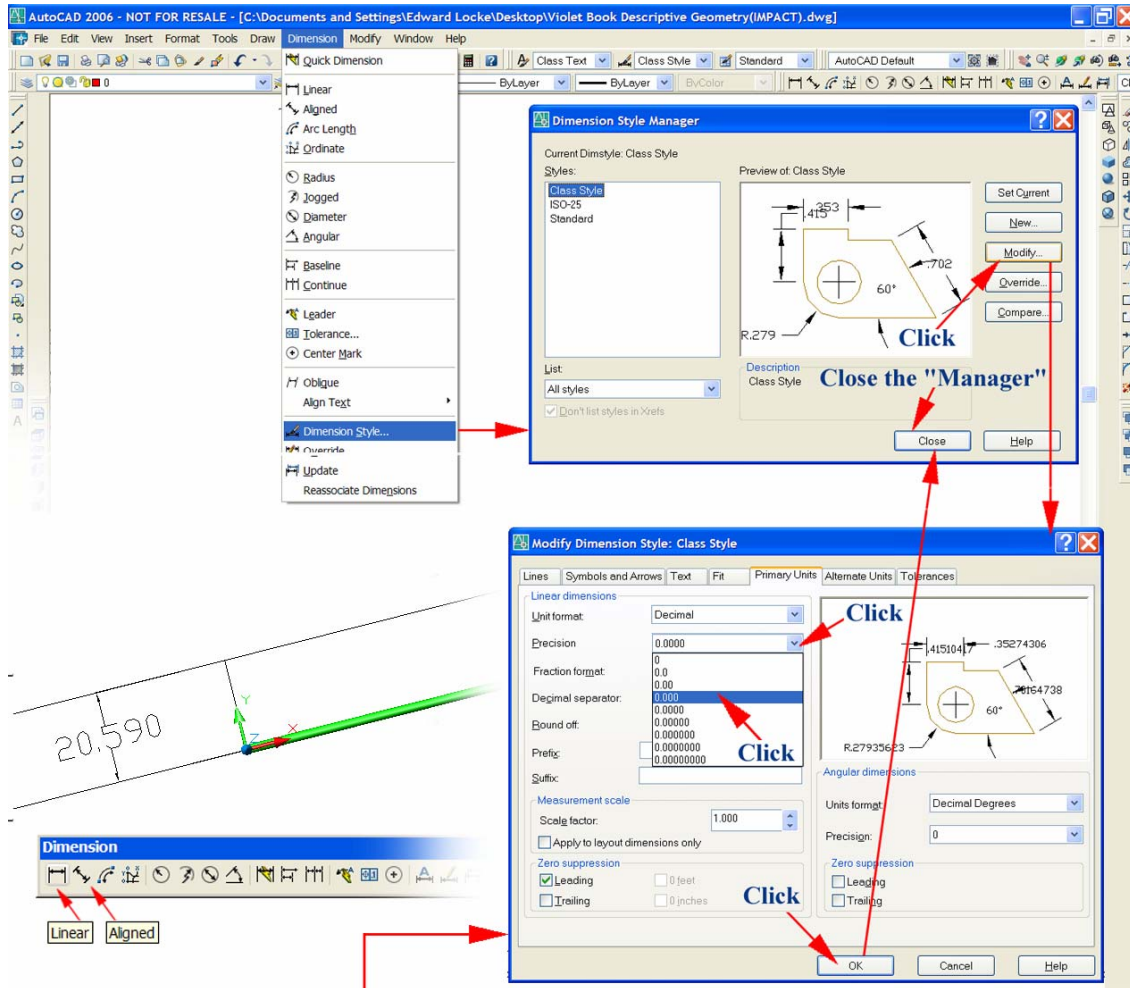
Figure 1C-2Q: Changing various dimension style and settings in the Primary Units tab of the Modify Dimension Style Window; and using either the Linear Dimension tool or the Aligned Dimension tool to measure the length of the shortest distance line (bottom-left).
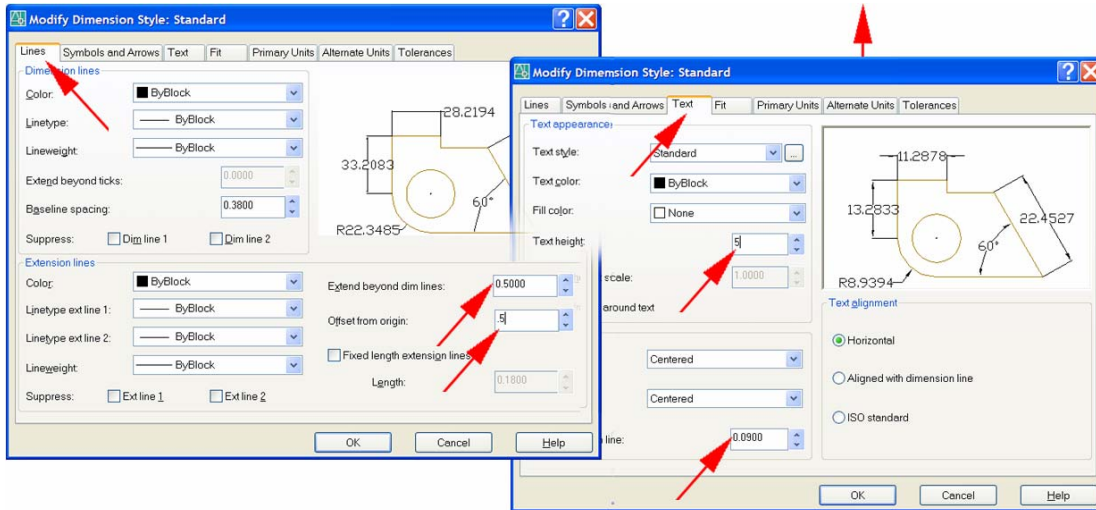
Figure 1C-2Q (Continued): Changing various dimension style and settings in the Lines (left) and Text tabs (right) of the Modify Dimension Style Window.

Placing the shortest distance line in the exact location where it connect the central axis lines of both pipes A-B and C-D with the help of construction lines

The shortest distance line created in the previous step is located at one end of the green pipe C-D. Now we need to find a method to place the shortest distance line at the exact location where it two ends can be connected to the central axis lines of both green pipe C-D and pink pipe A-B.
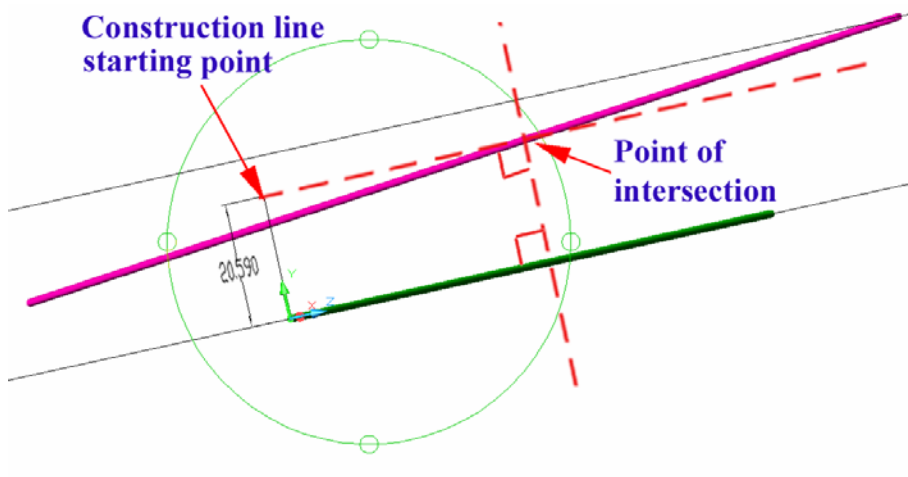

Figure 1C-2R: Visualizing a construction line parallel to the central axis of the green pipe C-D, staring from the top end point of the shortest distance line to intersect the central axis of the pink pipe A-B, and a perpendicular construction line through this point of intersection.
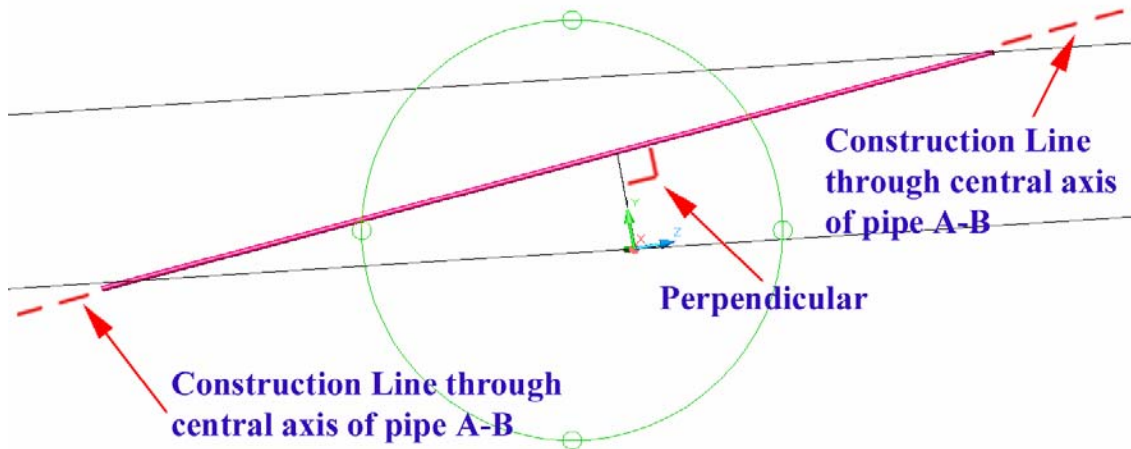
Figure 1C-2R (Cntinued): Rotating the model to the point view of the green pipe C-D (bottom).

Use the **3D Orbit** tool to rotate the model, as shown in *Figure 1C-2R* (top), and visualize the geometric relations among the two pipes and the shortest distance line. It is obvious that if we create a construction line parallel to the central axis of the green pipe C-D and starting at the top endpoint of the shortest distance line, then this parallel construction line will intersect the central axis line of the pink pipe A-B; in addition, since this new construction line is parallel to the central axis of the green pipe C-D, and the central axis of the green pipe C-D is perpendicular to the shortest distance line, another construction line perpendicular to both the construction line and the central axis of the green pipe C-D must be parallel to the shortest distance line. Using the **3D Orbit** tool to rotate the model such that the green pipe C-D is shown as a point view, as shown in *Figure 1C-2R* (top), the shortest distance line at one end of the green pipe C-D is perpendicular to the central axis of the pink pipe A-B.
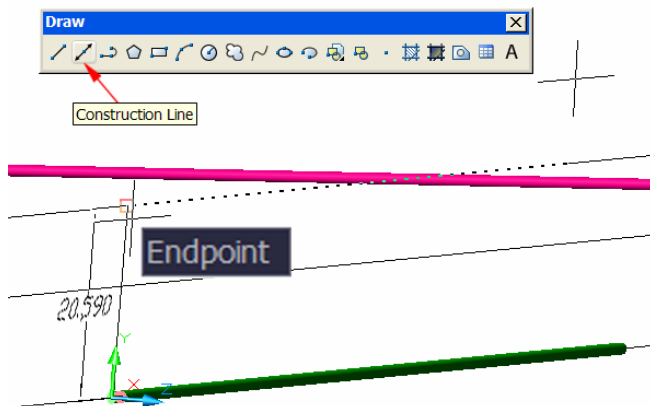

Figure 1C-2S: Creating the construction line parallel to the central axis of the green pipe C-D using the Construction Line tool, with the help of Endpoint Object Snap and the ORTHO.
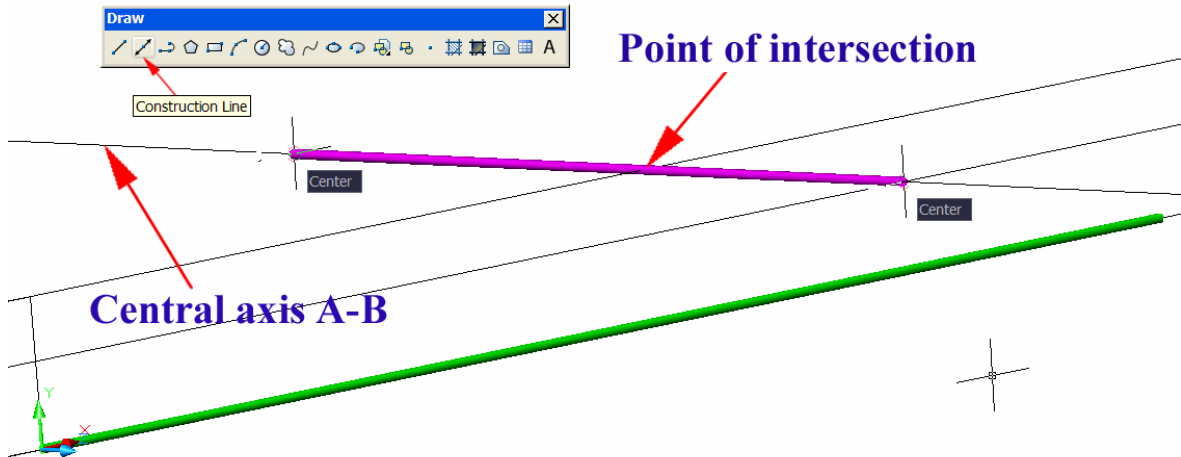
Figure 1C-2S (Continued): Creating the central axis line for the pink pipe A-B, using the Construction Line tool, with the help of Center Object Snap.

Use the **Construction Line** tool to create a construction line passing through the top endpoint of the shortest distance line, and parallel to the central axis line of the green pipe C-D, and the pink pipe A-D, as shown in *Figure 1C-2S* (top):

`Command: xl` [Type xl and press the **Enter** key to invoke the **Construction Line** tool]
`XLINE Specify a point or [Hor/Ver/Ang/Bisect/Offset]:` [Move the mouse-cursor close to the top end of the shortest distance line, and click once at the appearance of the **Endpoint** **Object Snap** indicator]
`Specify through point:  <Ortho on>` [With the help of the **ORTHO** constraint, drag the mouse-cursor in the direction parallel to the central axis of the green pipe C-D, and click once at any convenient location]
`Specify through point:` [Press the **Enter** key to exit the **Construction Line** tool. Press the **Enter** key to reactivate the **Construction Line** tool]

Next, use the **Construction Line** tool again to create central axis of red pipe, as shown in *Figure 1C-2S* (bottom):

`Command: xl XLINE Specify a point or [Hor/Ver/Ang/Bisect/Offset]:`
`Specify through point:  <Ortho on>` [Move the mouse-cursor close to one end of the pink pipe A-B, and click once at the appearance of the **Center** **Object Snap** indicator]
`Specify through point:` [Move the mouse-cursor close to the other end of the pink pipe A-B, and click once at the appearance of the **Center** **Object Snap** indicator]
`Specify through point:` [Press the **Enter** key to exit the **Construction Line** tool. Press the **Enter** key again to reactivate the **Construction Line** tool]
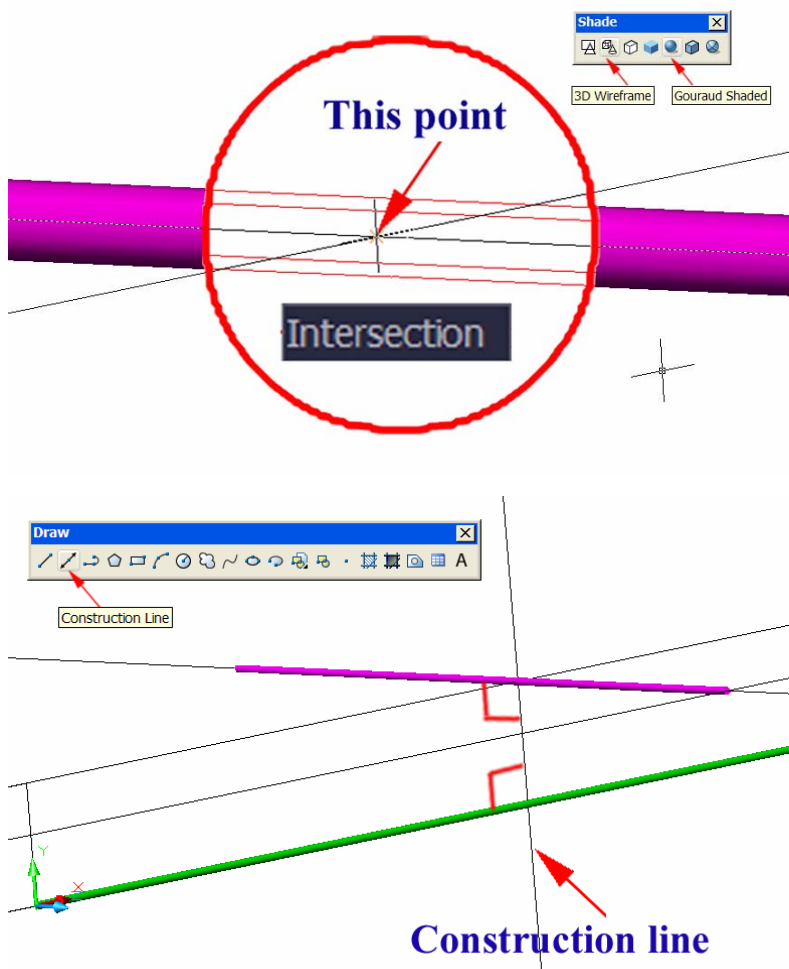
Figure 1C-2T: The point of intersection between the central axis of the pink pipe A-B and the construction line passing through the top endpoint of the existing shortest distance line, at one end of and parallel to the central axis of the green pipe C-D (top).

Creating the construction line perpendicular to the central axis construction line of the pink pipe A-B, and through the above-mentioned point of intersection (bottom).

The construction line parallel to the central axis line of the green pipe C-D intersects the central axis line of the pink axis, as shown in *Figure 1C-2T* (top). From this point of intersection, the shortest distance line connecting the central axis lines of both pipes A-B and C-D can be created. We can simply use the **Copy** tool to copy the existing shortest distance line from its top endpoint above the end of the green pipe C-D to the point of intersection (or we can use the **Move** tool to move it from its current location to the above-mentioned point of intersection); however, to offer a solid proof that the previous few steps make sense, and that the shortest distance line do actually connect the central axes of both pipes, we will take another approach, described as follows:

First, use the **Construction Line** tool to create a construction line, which is perpendicular to the above-mentioned point of intersection, and can be trimmed into the

shortest distance line connecting the central axes of both pipes A-B and C-D, as shown in
Figure 1C-2T (bottom):

```
Command: xl XLINE Specify a point or [Hor/Ver/Ang/Bisect/Offset]:
Specify through point:  <Ortho on> [Move the mouse-cursor close to the above
```
mentioned point of intersection, and click once at the appearance of the **Intersection**
Object Snap indicator]
```
Specify through point: [Press the Enter key to exit the Construction Line tool]
```
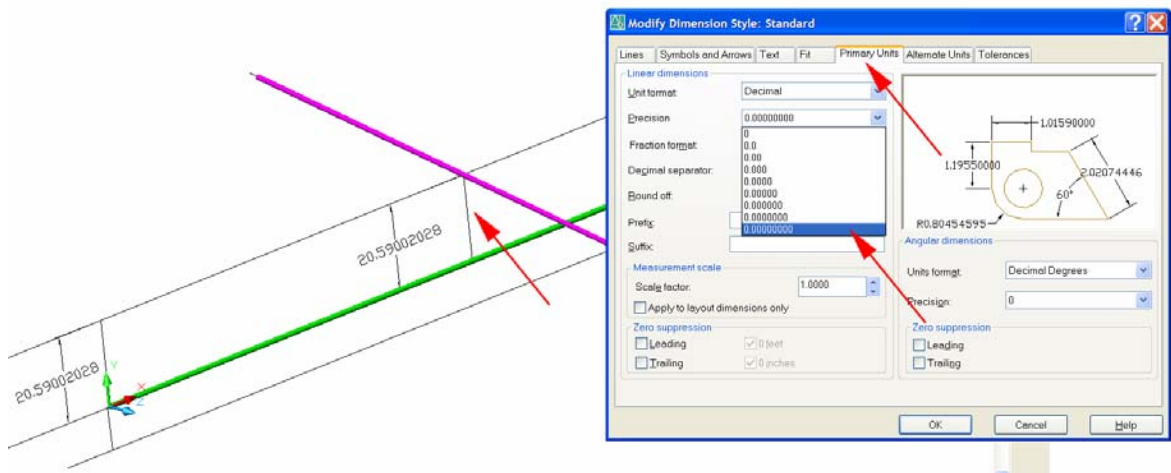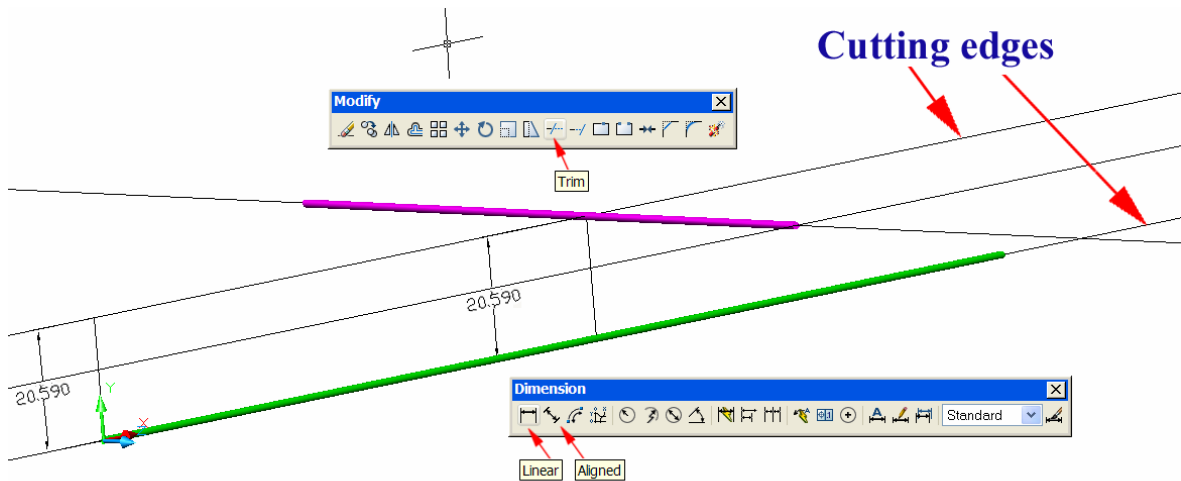

*Figure 1C-2U: Trimming off the top and bottom portions of the latest construction line to change it into the
shortest distance line (top). The same 20.59002028 value with the highest precision for the length of the two
shortest distance lines (bottom).*

Next, use the **Trim** tool to cut the latest construction line into the shortest distance
line, as shown in *Figure 1C-2U* (top):

```
Command: tr [Type tr and press the Enter key to invoke the Trim tool]
TRIM
View is not plan to UCS. Command results may not be obvious.
Current settings: Projection=UCS, Edge=None
Select cutting edges ...
Select objects or <select all>:
```
[Click-select the central axis line of the green pipe C-D]
```
1 found
Select objects:
```
[Click-select the central axis line of the pink pipe A-B]
```
1 found, 2 total
Select objects:
```
[Press the **Enter** key to end selection]
```
Select object to trim or shift-select to extend or
[Fence/Crossing/Project/Edge/eRase/Undo]:
```
[Move the mouse-cursor above construction line parallel to the central axis line of the green pipe C-D, and click on the latest construction line perpendicular to it. The top portion of the latest construction line disappears]
```
Select object to trim or shift-select to extend or
[Fence/Crossing/Project/Edge/eRase/Undo]:
```
[Move the mouse-cursor below the central axis line of the green pipe C-D, and click on the latest construction line perpendicular to it. The bottom portion of the latest construction line disappears]
```
Select object to trim or shift-select to extend or
[Fence/Crossing/Project/Edge/eRase/Undo]:
```
[Press the **Enter** key to exit the **Trim** tool]

In AutoCAD, if the "cutting edges" and the "objects" to be cut actually intersect, then the **Trim** tool can be used to trim the "objects" by the "cutting edges" directly, without using the additional option listed in the **Command Line** readout, such as **Project→View**, as previously explored in the beginning of this section. Such is the case with the latest use of the **Trim** tool, which has proved that the shortest distance line just created is connected to the central axis lines of both pink pipe A-B and green pipe C-D.

Next, use the **Linear Dimension** or **Aligned Dimension** tool to measure the length of the shortest distance line just created. Select either tool from the **Dimensions** tool bar by clicking on the tool icon; right-click to change the tool to a **Quick Dimension** setting; then click-select the shortest distance line; and then drag the mouse-cursor to a convenient location and click one to place the dimension. The dimension figure reads 20.590 (to the third digit after the decimal point, as set up in the **Modify Dimension Style** window, under the **Primary Units** tab and the in the **Precision** drop-down list), similar to the dimension obtained from the first shortest distance line created on one of the ends of the green pipe C-D. To prove that the length of both shortest distance lines are equal beyond doubt, go to the **Dimension→ Dimension Styles…** menu to open the **Dimension Styles Manager Dimension Styles** window; click the **Modify** button to open

the **Modify Dimension Style** window; select the **Primary Units** tab, click the ∨ button at
the right end of the **Precision** text field and scroll down the menu to select the
0.00000000 (the highest **Precision** setting with AutoCAD's **Dimension** tools); click the
OK button to return to the **Dimension Styles Manager** window, and click the **Close**
button to exit. The display of both dimension figures change to the same value of
20.59002028 for the length of the two shortest distance lines, as shown in *Figure 1C-2U*
(bottom). This proves that the length of the shortest distance is indeed precise.

       Press the **Ctrl** and z keys simultaneously to undo the change made in the
Precision drop-down list in the **Modify Dimension Style** window. The screen display of
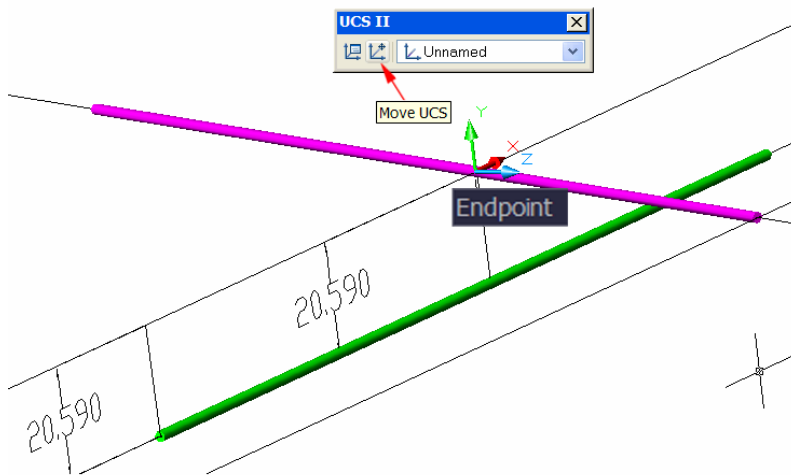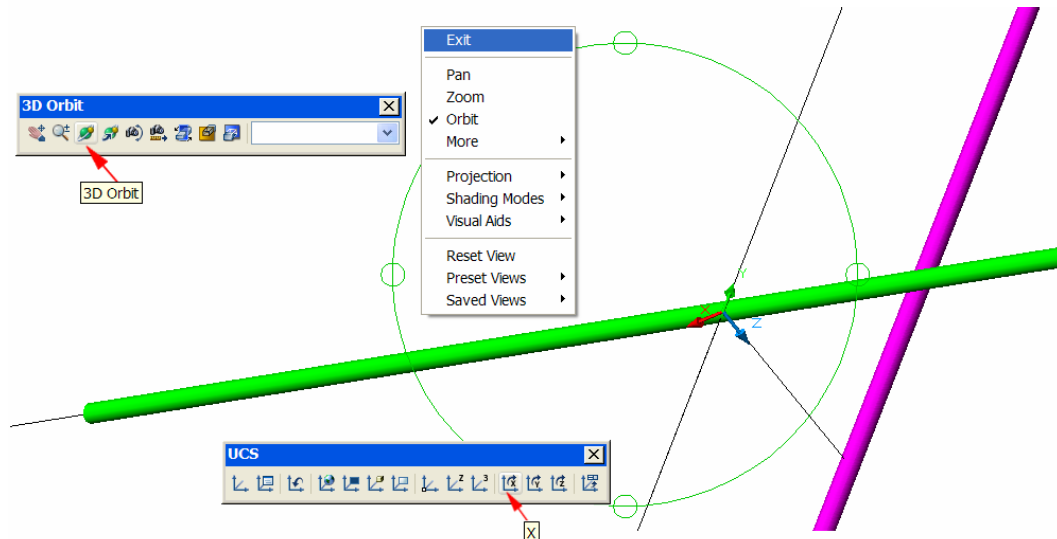the dimension figures return to the value of 20.590.

*Figure 1C-2V: Moving the
UCS to the end of the
shortest distance line at
the point of connection
with one pipe (top).
Rotating the UCS around
its x-axis to create a xy-
coordinate plane
perpendicular to the
shortest distance line
(bottom).*

Creating the shortest connection pipe

First, create a new **UCS** xy-coordinate plane perpendicular to the shortest distance line connecting the central axis lines of the pipes A-B and C-D. Since the current **UCS** at one end of the green pipe C-D has its y-axis parallel to the shortest distance line at the connection point, we only need to move it to one end of the this latest shortest distance line (*Figure 1C-2V*, top), and then rotate the **UCS** around the x-axis such that its z-axis will be aligned with the latest shortest distance line, and its xy-coordinate plane will be perpendicular to it (*Figure 1C-2V*, bottom-left):

Command: ucs [Type ucs and press the **Enter** key to invoke the **UCS** tool]
Current ucs name: *NO NAME*
Enter an option
[New/Move/orthoGraphic/Prev/Restore/Save/Del/Apply/?/World]
<World>: m [Type m and press the **Enter** key to select the **Move** option]
Specify new origin point or [Zdepth]<0,0,0>: [Move the mouse-cursor close to one end of the latest shortest distance line at the point of connection to both pipes A-B and C-D, and click once at the appearance of the **Endpoint** or **Intersection** **Object Snap** indicator. Press the **Enter** key to reactivate the **UCS** tool]
Command: ucs
Current ucs name: *NO NAME*
Enter an option
[New/Move/orthoGraphic/Prev/Restore/Save/Del/Apply/?/World]
<World>: x [Type x and press the **Enter** key to select the x-axis as the axis of rotation]
Specify rotation angle about X axis <90>: [Press the **Enter** key to accept the default 90 ° angle of rotation. The **UCS** has its z-axis aligned with the shortest distance line]
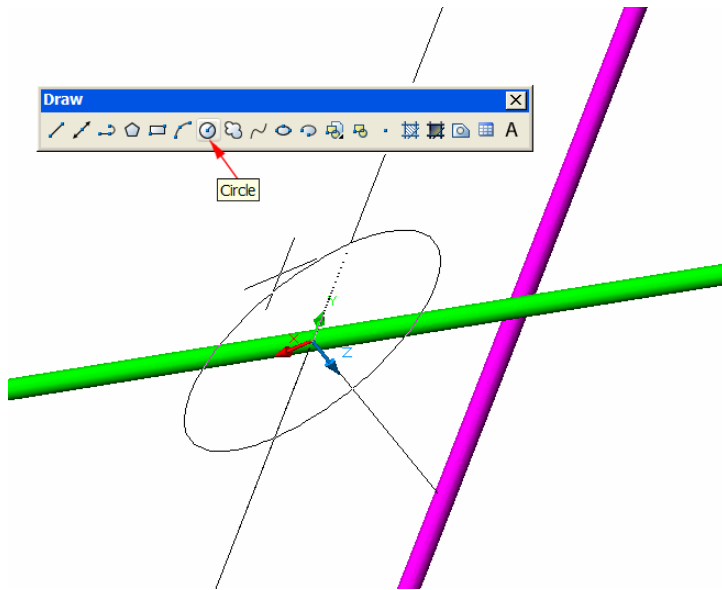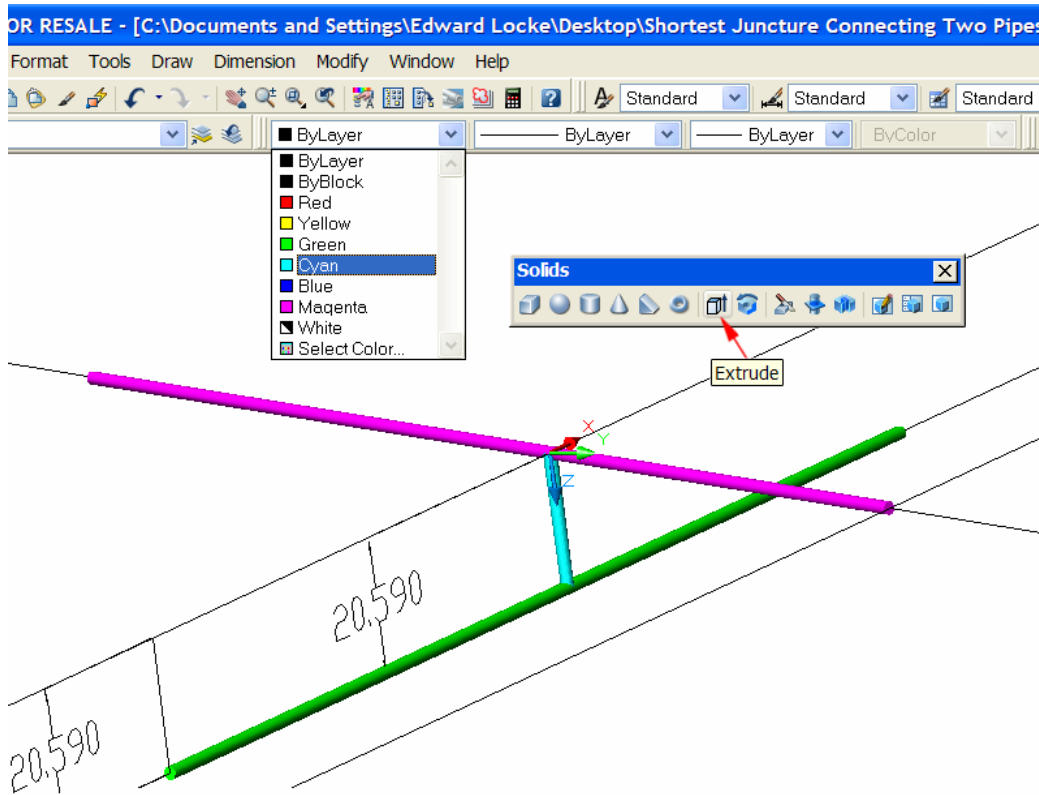
Figure 1C-2W: Creating the cross-section profile for the shortest connecting pipe.

The pipe A-B (in point view) cuts
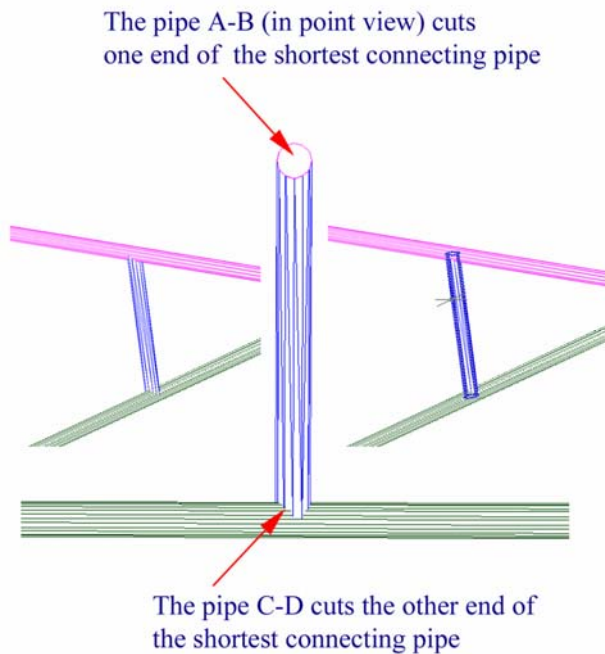one end of the shortest connecting pipe

*Figure 1C-2W (Continued): Extruding the
cross-section profile along the Path of the
shortest distance line into the shortest
connecting pipe; and changing its color (top).
Portions of the ends of the shortest connecting
pipe cut by the given pipes A-B and C-D
(bottom).*

The pipe C-D cuts the other end of
the shortest connecting pipe

Next, create the circular cross-section profile at the new **UCS xy-**coordinate plane
end of the latest shortest distance line, with the **Circle** tool (*Figure 1C-2V*, top):

Command: c [Type c and press the **Enter** key to invoke the **Circle** tool]
CIRCLE Specify center point for circle or [3P/2P/Ttr (tan tan
radius)]: 0,0,0 [Type 0,0,0 and press the **Enter** key to set the center of the circle at
the **Origin** of the new **UCS**]
Specify radius of circle or [Diameter] <1.0000>: d [Type d and press the
Enter key to select the **Diameter** option]
Specify diameter of circle <2.0000>: 2 [Type 2 and press the **Enter** key to set
the **Diameter** of the circle as 2 inches]

Next, use the **Extrusion** tool to extrude the circular end cross-section profile
along the **Path** of the shortest distance line into a three-dimensional pipe (*Figure 1C-2V*,
bottom):

Command: ext [Type ext and press the **Enter** key to invoke the **Extrusion** tool]
EXTRUDE
Current wire frame density: ISOLINES=4
Select objects: [Click-select the circular cross-section profile just created]
1 found
Select objects: [Press the **Enter** key to end selection]

`Specify height of extrusion or [Path]:` p [Type p and press the **Enter** key to select the **Path** option]
`Select extrusion path or [Taper angle]:` [Click-select the shortest distance line connected to the central axes of both pipes. The shortest connection pipe appears on the screen in black color]

    The basic three-dimensional model of the shortest connecting pipe is completed. When we use the term "basic" here, we mean that the length of the shortest connecting pipe is the "basic" size to work with (in other words, the size of the pipe to be cut initially). In reality, the actual size of the some portions, around the intersection curves, at the ends of the pipe are slightly smaller then the "basic" size, since the "basic" length is measured along the central axis; however, the diameters of both given pipes A-B and C-D naturally cut down portions of the connecting pipe (as shown in *Figure 1C-2W*, bottom-right). This technical detail is however beyond the scope of this Module.

Determining the geometric properties of the pipes

    We will now use the **Locate Point** tool to identify the centers of both end cross-sections of each pipe; and use the **Multiline Text** tool to label each point. In addition, we will use the **Distance** tool to measure the length of each pipe (*Figure 1C-2X*).

    Before we start, we need to return the **UCS** to its default **World** setting since in the beginning of the project, all **Absolute Coordinate Entry** points are defined in AutoCAD's default **World** setting. If we fail to do this, then the results we will obtain by using the **Locate Point** tool will not be correct.

    To identify the centers of both end cross-sections of each pipe, use the **Locate Point** tool as follows, for each center point of the end cross-section of the pipes:

`Command:` id [Type id and press the **Enter** key to invoke the **Locate Point** tool]
`Specify point:` cen of [Type cen and press the **Enter** key to select the **Center** as the **Object Snap**; move the mouse-cursor closer to the end of the pipe, and click once at the appearance of the **Center Object Snap** indicator]
`X = -110.0000        Y = -30.0000       Z = 110.0000`
[The **Command Line** displays the (x, y, z) coordinates of the point]

    The (x, y, z) coordinates of the endpoints of the pipes A-B and C-D identified with the **Locate Point** tool, and their **Command Line** displays match the given conditions, and are listed as follows:

Point A:
```
Command: id Specify point: X = -40.0000  Y = 90.0000  Z = -10.0000
```

Point B:
```
Command: id Specify point: X = -10.0000  Y = -4.0000  Z = 160.0000
```

Point C:
```
Command: id Specify point: X = -110.0000 Y = -30.0000 Z = 110.0000
```

Point D:
```
Command: id Specify point: X = 60.0000   Y = 60.0000  Z = 50.0000
```

The (x, y, z) coordinates of the points of connection between the shortest connecting pipe and the pipes A-B and C-D identified with the **Locate Point** tool, and their **Command Line** displays are listed as follows:

Point x: The point of connection between the shortest connecting pipe and the pipes A-B
```
Command: id Specify point: X = -22.7657  Y = 35.9993   Z = 87.6609
```
Point y: The point of connection between the shortest connecting pipe and the pipes C-D
```
Command: id Specify point: X = -17.4207  Y = 19.0126   Z = 77.3250
```
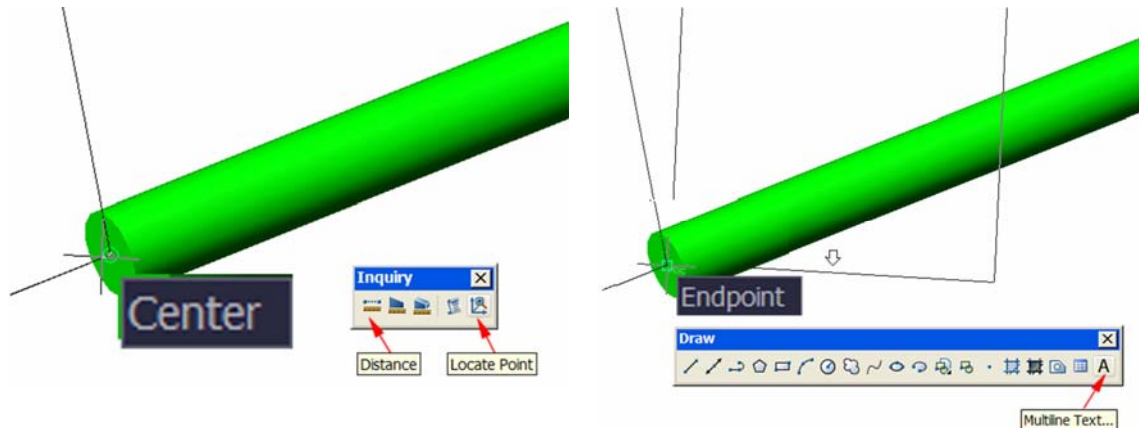

Figure 1C-2X: The Locate Point and Distance tools in the Inquiry tool bar (left). The Multiline Text tool (right).
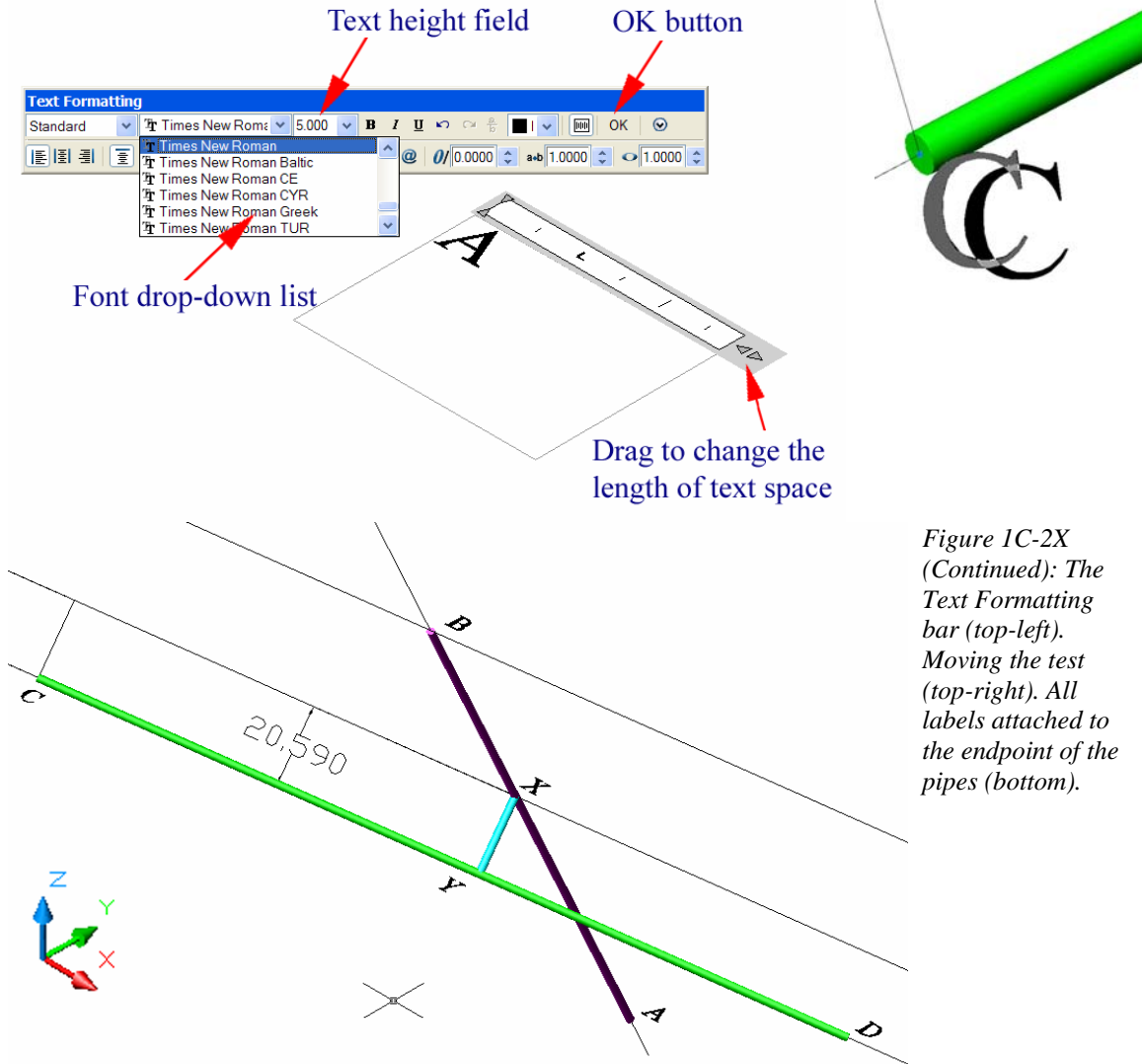
Figure 1C-2X (Continued): The Text Formatting bar (top-left). Moving the test (top-right). All labels attached to the endpoint of the pipes (bottom).

After identifying each point, use the **Multiline Text** tool to place a label next to it To use this tool, click its tool icon on the **Draw** tool bar (or type mt and press the **Enter** key to invoke the tool); move the mouse-cursor closer to the endpoint of the pipe, and click once when the **Object Snap** (**Endpoint** and **Center**) indicator appears (*Figure 1C-2X*, top right).; a flashing cursor appears; click once and drag-out for a text space; the **Text Formatting** bar appears; type 5 in the text height field and select a font from the drop-down list; start typing the text, and then click the **OK** button the complete (*Figure 1C-2X*, middle-left). If the text is too close to the end of the pipe, then click-select it; hold the left mouse button, drag it away to a better location, and release the mouse button to

finish (*Figure 1C-2X*, middle-right). Use the **3d Orbit** and **Zoom** tools to help visualizing the position of the text with respect to the pipes if necessary.

Next, use the **Distance** tool to find the length of the pipe. For the pipe A-B, the length given by the tool is 196.5604, which matches the calculation we have:

`Command: di` [Type di and press the **Enter** key to invoke the **Distance** tool]
`DIST Specify first point:` [Move the mouse-cursor close to one end of the pink pipe A-B, and click once at the appearance of the **Center** **Object Snap** indicator]
`Specify second point:` [Move the mouse-cursor close to the other end of the pink pipe A-B, and click once at the appearance of the **Center** **Object Snap** indicator]
`Distance = 196.5604, Angle in XY Plane = 108, Angle from XY Plane = 300`
`Delta X = -30.0000, Delta Y = 94.0000, Delta Z = -170.0000`
[The **Command Line** displays the related information, including the length 196.5604]

The calculation using math formula yields the same result:

$$D_{AB} = \sqrt{(\Delta X_{AB})^2 + (\Delta Y_{AB})^2 + (\Delta Z_{AB})^2} = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (Z_B - Z_A)^2}$$
$$= \sqrt{[(-10)-(-40)]^2 + [(-4)-(90)]^2 + [(160)-(-10)]^2}$$
$$= \sqrt{38636} \approx 196.5604233 = 196.56\,[\text{in}]$$

For the pipe C-D, The **Command Line** displays the following information:

`Distance = 201.4944, Angle in XY Plane = 28, Angle from XY Plane = 343`
`Delta X = 170.0000,  Delta Y = 90.0000,   Delta Z = -60.0000`
, including the length 196.5604]

The calculation using math formula yields the same result:

$$D_{CD} = \sqrt{(\Delta X_{CD})^2 + (\Delta Y_{CD})^2 + (\Delta Z_{CD})^2} = \sqrt{(X_D - X_C)^2 + (Y_D - Y_C)^2 + (Z_D - Z_C)^2}$$
$$= \sqrt{[(60)-(-110)]^2 + [(60)-(-30)]^2 + [(50)-(110)]^2}$$
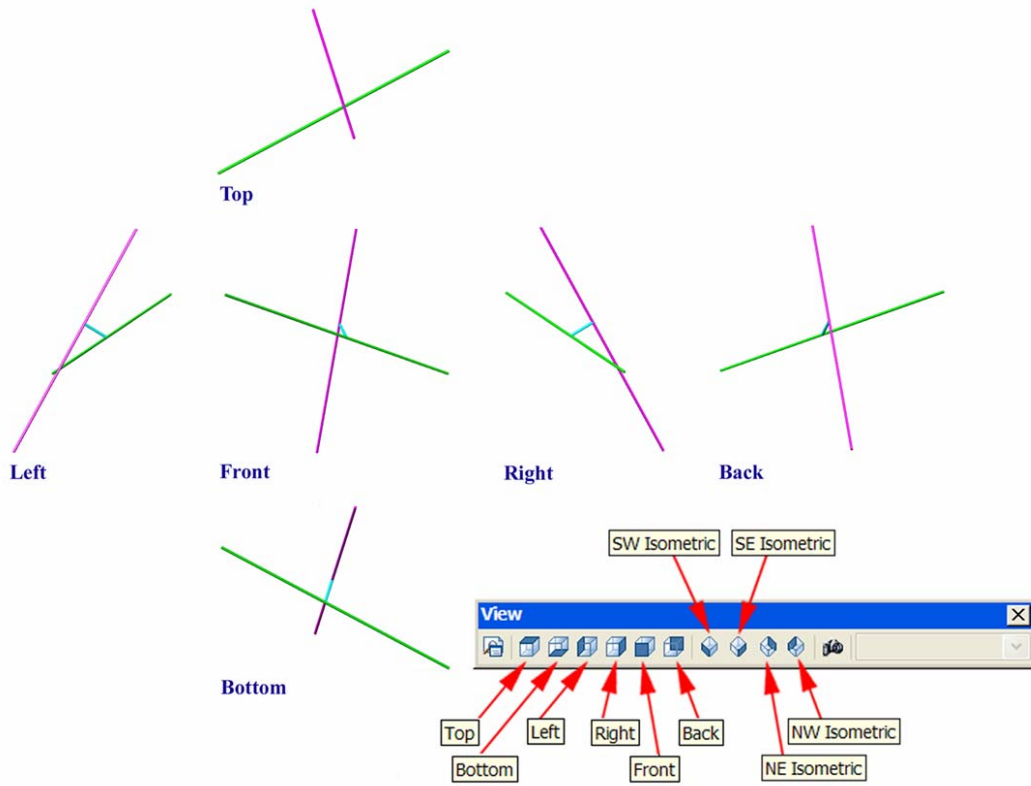$$= \sqrt{40600} \approx 201.4944168 = 201.49\,[\text{in}]$$

Figure 1C-2Y: The orthographic view son the screen obtained by the View tools.
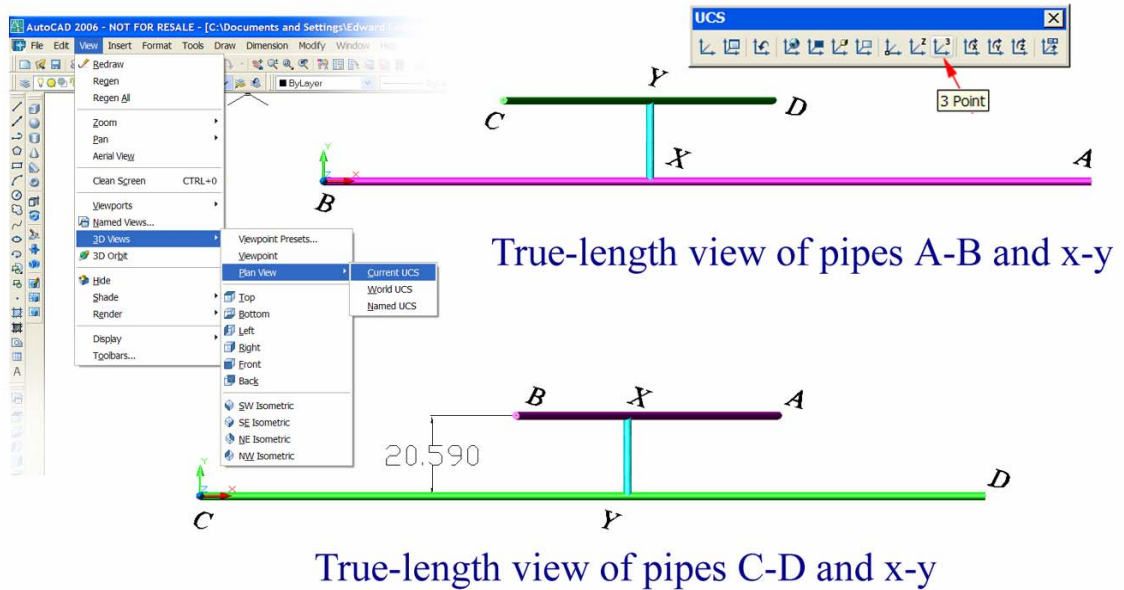

Figure 1C-2Z: Obtaining the true-length views of the pipes.

Obtaining the screen views of the pipes

To obtain orthographic view on the screen, click the tool icons on the **View** tool bar (*Figure 1C-2Y*). To obtain the true-length view of the pipes, first use the **UCS** tool with the **3 Point** option to establish a xy-plane containing the central axes of the pipes; then go to the **View→3D View→Plan View→ Current UCS** menu to obtain the true-length view perpendicular to the line of sight.

The Section is now complete. Save and close the file.

Section 3
Solving the Problems of the Shortest Distance between Two Lines with Mathematical skills

We will now try to solve the same descriptive geometry problem listed in Section 17 with mathematical skills, mainly, the skills taught in a typical college beginning and intermediate calculus courses; and we will then compare the calculated result with the result obtained from AutoCAD three-dimensional modeling and measurement, and find the % error between the two valued with the calculated value taken as the theoretical value. We will proceed to solve the problem in a typical math problem solution process.
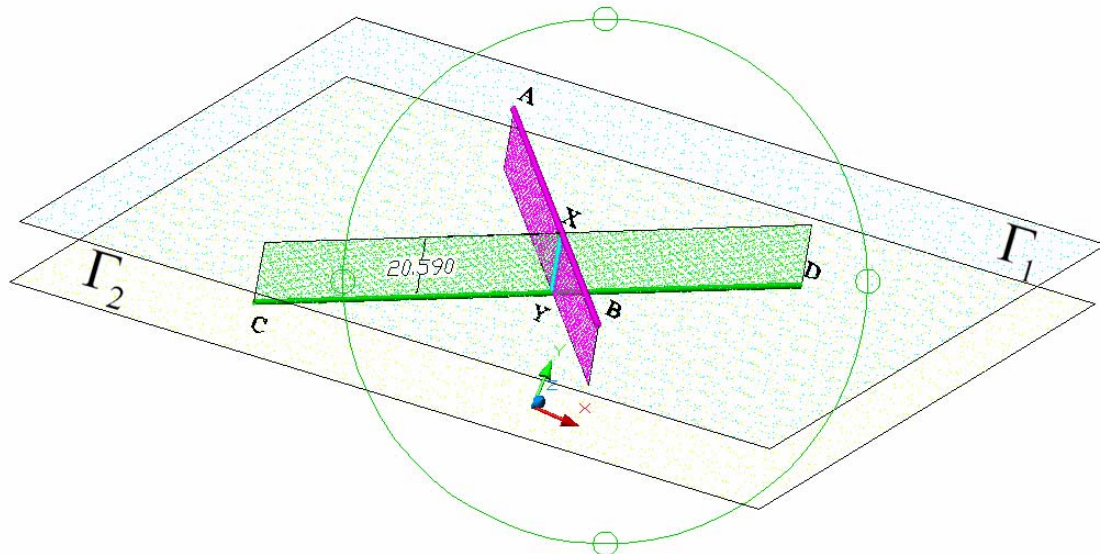

Figure 1C-3A: The green and red pipes and the planes 1 and 2.

Given:
The given conditions for the central lines of the two pipes

- The coordinates for the central axis line A-B of the Pipe A-B:
  Point A: (-40, 90, -10).
  Point B: (-10,-4,160).

- The coordinates for the central axis line C-D of the Pipe C-D:
  Point C: (-110,-30,110).
  Point D: (60,60,50).

Find:
The shortest distance between the two central lines of the pipes A-B and C-D.

Solution:

Let the central axis line for the pipe A-B be denoted as L_1; and let the central axis line for the pipe C-D be denoted as L_2.

Let the plane containing the line L_1 be denoted as Γ_1; and let the plane containing the line L_2 be denoted as Γ_2.

<u>For the central axis line A-B for the pipe A-B (denoted as L_1):</u>

Given the points A(-40, 90, -10) and B(-10, -4, 160), the direction vector is
$$\vec{v}_1 = \overrightarrow{AB} = \langle -10 - (-40), -4 - 90, 160 - (-10) \rangle = \langle 30, -94, 170 \rangle$$

Therefore, the direction numbers for the direction vector $\vec{v}_1$ are $\begin{cases} a = 30 \\ b = -94 \\ c = 170 \end{cases}$.

Let A(-40, 90, -10) be $P_0(x_0, y_0, z_0)$, then $\begin{cases} x_0 = -40 \\ y_0 = 90 \\ z_0 = -10 \end{cases}$; and

The parametric equations for L_1 are $\begin{cases} x = x_0 + at = -40 + 30t \\ y = y_0 + bt = 90 - 94t \\ z = z_0 + ct = -10 + 170t \end{cases}$

For the central axis line C-D for the pipe C-D (denoted as L_2):

Given the Points C(-110, -30, 110) and D(60, 60, 50), the direction vector is
$$\vec{v}_2 = \overrightarrow{AB} = \langle 60 - (-110), 60 - (-30), 50 - 110 \rangle = \langle 170, 90, -60 \rangle$$

Therefore, the direction numbers for the direction vector $\vec{v}_2$ are $\begin{cases} a = 170 \\ b = 90 \\ c = -60 \end{cases}$.

Let C(-110, -30, 110) be $P_0(x_0, y_0, z_0)$, then $\begin{cases} x_0 = -110 \\ y_0 = -30 \\ z_0 = 110 \end{cases}$; and

The parametric equations for L_2 are $\begin{cases} x = x_0 + as = -110 + 170s \\ y = y_0 + bs = -30 + 90s \\ z = z_0 + cs = 110 - 60s \end{cases}$

The relative positions of the two lines L_1 and L_2 in three-dimensional space can be parallel, intersecting, or skew.

Checking for parallelism:

Since the components of their corresponding direction vectors $\vec{v}_1 = \langle 30, -94, 170 \rangle$ and $\vec{v}_2 = \langle 170, 90, -60 \rangle$ are not multiples by a common factor:
$$\left. \begin{array}{l} \vec{v}_1 = \langle 30, -94, 170 \rangle \\ \vec{v}_2 = \langle 170, 90, -60 \rangle \end{array} \right\} \Rightarrow \langle 30, -94, 170 \rangle \neq k \langle 170, 90, -60 \rangle \Rightarrow \vec{v}_1 \neq k\vec{v}_2$$

Therefore, their corresponding direction vectors are not parallel; and the two lines L_1 and L_2 are not parallel.

Checking for intersection:

If the two lines L_1 and L_2 intersect each other, then they must share one point in common; or in other words, both lines must contain one point with the same (x, y, z) coordinates.

Setting the (x, y, z) coordinates in the sets of parametric equations of both lines L_1 and L_2 equal, we have

$$L_1 : \begin{cases} x = -40 + 30t \\ y = 90 - 94t \\ z = -10 + 170t \end{cases} \overset{?}{=} L_2 : \begin{cases} x = -110 + 170s \\ y = -30 + 90s \\ z = 110 - 60s \end{cases}$$

$$\begin{cases} -40 + 30t = -110 + 170s & (1) \\ 90 - 94t = -30 + 90s & (2) \\ -10 + 170t = 110 - 60s & (3) \end{cases}$$

Using the equation (1) and (2) to solve for t and s:

$$\left. \begin{array}{l} -40 + 30t = -110 + 170s \quad (1) \\ 90 - 94t = -30 + 90s \quad\quad (2) \end{array} \right\} \rightarrow$$

$$t = \frac{-110 + 170s + 40}{30} = \frac{-70 + 170s}{30} = \frac{-7 + 17s}{3} \quad (4)$$

$$90 - 94\left(\frac{-7 + 17s}{3}\right) = -30 + 90s \quad \rightarrow \quad 90 - (94)\left(\frac{-7}{3}\right) - (94)\left(\frac{17s}{3}\right) = -30 + 90s \quad \rightarrow$$

$$-(94)\left(\frac{17s}{3}\right) - 90s = -30 - 90 + (94)\left(\frac{-7}{3}\right) \quad \rightarrow \quad -\frac{1598s}{3} - \frac{270s}{3} = -\frac{(120)(3)}{3} - \frac{658}{3} \quad \rightarrow$$

$$+\frac{1868s}{3} = +\frac{1018}{3} \quad \rightarrow \quad s = \frac{1018}{1868} \approx 0.545 \quad (4); \quad (5) \rightarrow (1):$$

$$-40 + 30t = -110 + 170\left(\frac{1018}{1868}\right) \quad \rightarrow$$

$$t = \frac{-110 + 170\left(\frac{1018}{1868}\right) + 40}{30} = \frac{-\frac{(70)(1868)}{1868} + \frac{173060}{1868} + \frac{(40)(1868)}{1868}}{30}$$

$$t = \frac{-(70)(1868) + 173060 + (40)(1868)}{(30)(1868)} = \frac{-130760 + 173060 + 74720}{56040} = \frac{117020}{56040} \approx 2.088 \quad (6)$$

Substituting $s = 0.545$ and $t = 2.088$ into equation (3):

$$-10 + 170t \overset{?}{=} 110 - 60s$$

$$-10 + (170)(2.088) \overset{?}{=} 110 - (60)(0.545)$$

$$344.96 \neq 77.3 \quad (3)$$

The substitution fails to satisfy the condition of the equation (3). Therefore, No common point can be found on both lines L_1 and L_2; and L_1 and L_2 do not intersect.

Therefore, the two lines L_1 and L_2 are skew. In the 3-space, if two lines are skew, they do not intersect and are not parallel (and therefore, do not lie on the same plane); instead, they must lie on two parallel planes.

Next, find the equations for the two parallel planes Γ_1 and Γ_2, which contain the lines

$$L_1 : \begin{cases} x = -40 + 30t \\ y = 90 - 94t \\ z = -10 + 170t \end{cases} \quad \text{and} \, L_2 : \begin{cases} x = -110 + 170s \\ y = -30 + 90s \\ z = 110 - 60s \end{cases}$$

Since the two lines L_1 and L_2 are skew, they can be viewed as lying on two parallel planes Γ_1 and Γ_2; and the distance between L_1 and L_2 equals the distance between Γ_1 and Γ_2, which can be computed by using the formula $D = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$

The common normal to both planes Γ_1 and Γ_2 must be orthogonal to both $\vec{v}_1 = \langle 30, -94, 170 \rangle$ (the direction vector of L_1) and $\vec{v}_2 = \langle 170, 90, -60 \rangle$ (the direction vector of L_2), therefore, the normal vector perpendicular to both planes and parallel to the shortest distance line is the cross product of the direction vectors of the two lines $\vec{v}_1 = \langle 30, -94, 170 \rangle$ and $\vec{v}_2 = \langle 170, 90, -60 \rangle$:

$$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 30 & -94 & 170 \\ 170 & 90 & -60 \end{vmatrix} = \begin{vmatrix} -94 & 170 \\ 90 & -60 \end{vmatrix} \hat{i} - \begin{vmatrix} 30 & 170 \\ 170 & -60 \end{vmatrix} \hat{j} + \begin{vmatrix} 30 & -94 \\ 170 & 90 \end{vmatrix} \hat{k}$$

$$\vec{n} = [(-94)(-60) - (170)(90)]\hat{i} - [(30)(-60) - (170)(170)]\hat{j} + [(30)(90) - (-94)(170)]\hat{k}$$

$$\vec{n} = [5640 - 15300]\hat{i} - [-1800 - 28900]\hat{j} + [2700 - (-15980)]\hat{k}$$

$$\vec{n} = -9660\hat{i} + 30700\hat{j} + 18680\hat{k}$$

The direction numbers for the normal vector $\vec{n}$ are $\begin{cases} a = -9660 \\ b = 30700 \\ c = 18680 \end{cases}$

Set $s = 0$ in the equations of the line L_2:

$$(x_0, y_0, z_0) \in L_2 : \begin{cases} x_0 = -110 + 170s_0 = -110 + 170(0) = -110 \\ y_0 = -30 + 90s_0 = -30 + 90(0) = -30 \\ z_0 = 110 - 60s_0 = 110 - 60(0) = 110 \end{cases}$$

We get the point $(x_0, y_0, z_0) = (-110, -30, 110)$ on the line L_2, and so the equation for the plane Γ_2 is computed to be $-9600x + 30700y + 18680z - 2189800 = 0$, as follows:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$
$$(-9660)[x - (-110)] + (30700)[y - (-30)] + (18680)(z - 110) = 0$$
$$(-9600)(x + 110) + (30700)(y + 30) + 18680z - 2054800 = 0$$
$$-9600x - 1056000 + 30700y + 921000 + 18680z - 2054800 = 0$$
$$-9600x + 30700y + 18680z - 1056000 + 921000 - 2054800 = 0$$
$$-9600x + 30700y + 18680z - 2189800 = 0$$

Set $t = 0$ in the equations for the line L_1:
$$(x_0, y_0, z_0) \in L_1 : \begin{cases} x_0 = -40 + 30t = -40 + (30)(0) = -40 \\ y_0 = 90 - 94t = 90 - (94)(0) = 90 \\ z_0 = -10 + 170t = -10 + (170)(0) = -10 \end{cases}$$

We get the point $(x_0, y_0, z_0) = (-40, 90, -10)$ on the line Γ_1, and so the distance between L_1 and L_2 is the same as the distance from the point $(-40, 90, -10)$ to the plane Γ_2 with the equation of $-9600x + 30700y + 18680z - 2189800 = 0$.

The direction numbers a, b, and c, and $d = -(ax_0 + by_0 + cz_0)$ for the equation of the

plane Γ_2 are: $\begin{cases} a = -9600 \\ b = 30700 \\ c = 18680 \\ d = -2189800 \end{cases}$

The distance can then be computed by using the formula

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|(-9600)(-40) + (30700)(90) + (18680)(-10) + (-2189800)|}{\sqrt{(-9600)^2 + (30700)^2 + (18680)^2}}$$

$$D = \frac{|384000 + 2763000 - 186800 - 2189800|}{\sqrt{92160000 + 942490000 + 348942400}} = \frac{|770400|}{\sqrt{1383592400}} = 20.71153038 \approx 20.712 \ in$$

The value for the shortest distance D between the two pipes obtained from calculation with calculus skills is 20. 71153038. Compared with the value obtained in AutoCAD 3D modeling and measurement (20.590), the discrepancy is rather small (0.58%).

$$\% \ Error = \frac{|D_{Theoretical} - D_{Experiment}|}{D_{Theoretical}} \times 100\%$$

$$\% \ Error = \frac{|D_{Calculus} - D_{AutoCAD}|}{D_{Calculus}} \times 100\% = \frac{|20.712 - 20.590|}{20.712} \times 100\% \approx 0.59\%$$

Section 4
Determining the Shortest Distance between a Line and a Point
(Given: Two Adjacent Views, Both Showing the Line as Foreshortened)

Basic definition of the distance between a point and a line in three-dimensional space

When we talk about the distance between a line and a point, we mean the shortest distance. The shortest distance between a point and a line can be found by drawing a construction line from the point and perpendicular to the line. The skill to find the shortest distance between a point and a line is especially important in mining and civil engineering.

Basic strategy for finding the shortest distance between a point and a line

To start, in the *Points, Lines, & Planes in the Three-Dimensional Space.dwg* file (inside the *Given Problems* folder, in the *Student Descriptive Geometry Learning Resource CD*); use the **Zoom** tool to zoom in the area under the tile of *Determining the Shortest Distance between A Line and A Point (Given: Two Adjacent Views, Both Showing the Line as Foreshortened)*; and keep the **ORTHO** and **OTRACK** buttons beneath the **Command Line** turned on, and the **Node** running **Object Snap** in the **Drafting Settings** window turned on.

Two adjacent views are needed in this type of project. In this *Section*, two adjacent views (the top view and the front view) of a line and a point are given, both showing the line as foreshortened, as shown in *Figure 1C-4A*. The key to finding the shortest distance between the given point and line is to draw an auxiliary view (point view) of the line, which turns the view of the line from its foreshortened view to a point view, and allows the measurement of the shortest distance between the given point and the line (which appears as a point in its point view).
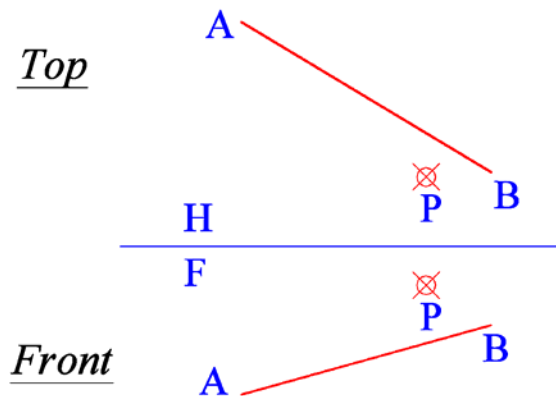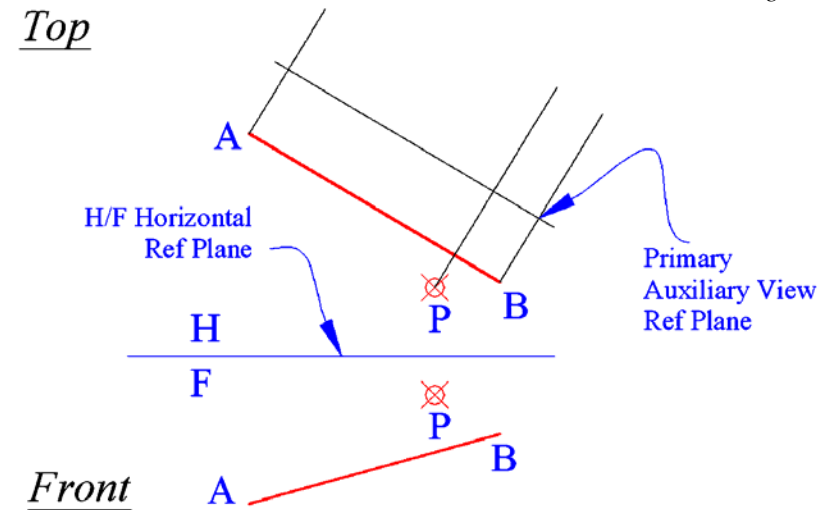

Figure 1C-4A: The given views.

Figure 1C-4B: Starting the primary auxiliary view.

A primary auxiliary view is needed to first find the true-length view of the line; and then a secondary auxiliary view is projected upon the primary auxiliary view to find the point view of the line, and the shortest distance between the line and the point is measured in the secondary auxiliary view. The step-by-step procedures are explained as follows:

Creating the First Auxiliary view (the true-length view of the line):

First, start to create the Primary Auxiliary view, select a view where the line is shown as foreshortened (we select the Top view; but selecting the Front view will also work, since both top and front views show the given line as foreshortened). We may use the **UCS** tool with **3-Point** option to align the **UCS** with line A-B in the top view; turn **ORTHO** on; then draw the projection lines from the points A, B, and P, as well as the Primary Auxiliary View Ref Plane line as shown in *Figure 1C-4B*. We have now obtained, from the Top view, the first set of dimensions. We will obtained the second set of dimensions, which are perpendicular to the first set of dimensions, from the view adjacent to the view where we have obtained the first set of dimensions, which is the Front view.
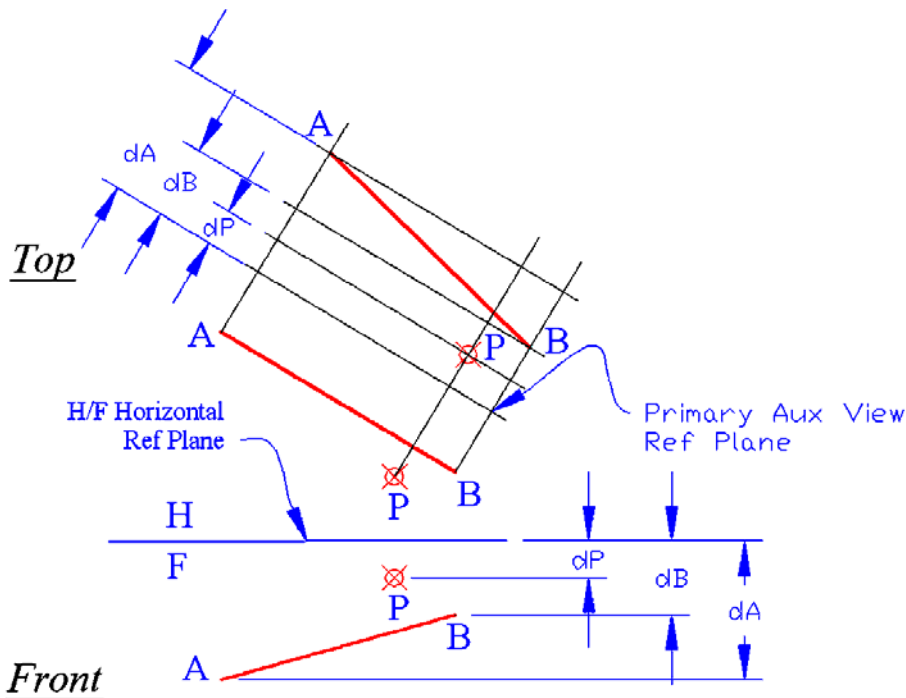

Figure 1C-4C: Measuring the distances in the Front view and setting the same distances off the Ref Plane Line in the Primary Auxiliary view.
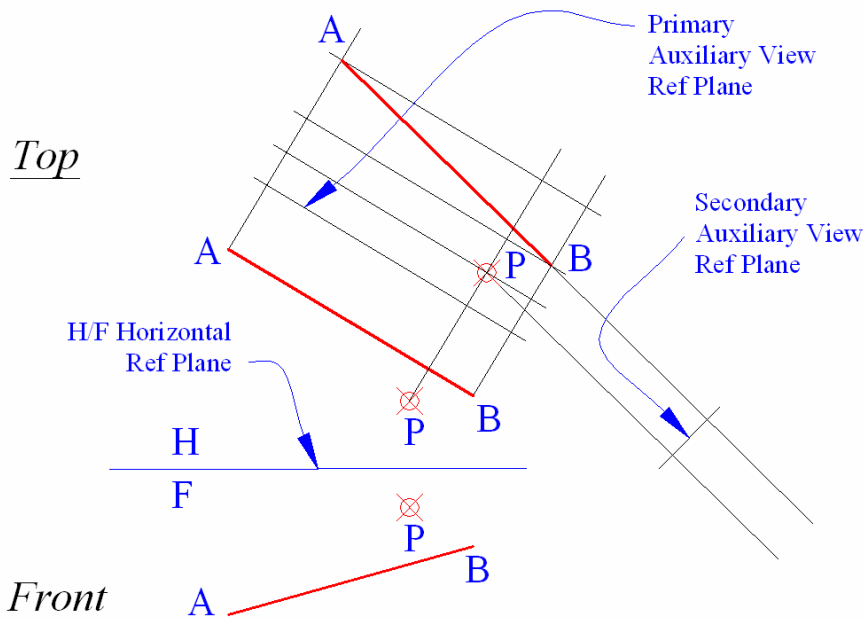
Figure 1C-4D: Starting projection lines for the Secondary Auxiliary view.

Return **UCS** to **Word** settings; and in the Front view, use the **Linear Dimension** tool to measure the distance from the H/F Horizontal Ref Plane line to the point P, and the endpoints of the foreshortened line B and A (dP, dB, and dA). Next, in the Primary Auxiliary view, use the **Offset** tool to set off construction lines at distances dP, dB, and dA from the Primary Auxiliary View Ref Plane line. Next, cross-referencing between the Front view and the Top view, determine the points P, A and B in the Primary Auxiliary view; draw line A-B and point P (*Figure 1C-4C*).

Creating the Secondary Auxiliary view (the endpoint view of the line):

Next, start to create the Secondary Auxiliary view. Align the **UCS** with line A-B in the Primary Auxiliary view; turn **ORTHO** on; draw projection lines from the points P and B into the Secondary Auxiliary view; draw the Secondary Auxiliary View Ref Plane line (*Figure 1C-4D*). We have now obtained, from the Primary Auxiliary view, the first set of dimensions. We will obtained the second set of dimensions, which are perpendicular to the first set of dimensions, from the view adjacent to the view where we have obtained the first set of dimensions, which is the Top view.

Determining the shortest distance between the given point and line:

Use the **UCS** tool with **3-Point** option to align the **UCS** with line A-B in the Top view, and then use the **Linear Dimension** tool to measure the distances from the Primary Auxiliary View Ref Plane line to point P and the endpoints of line A-B in the Top view (dP and dAB; next, use the **Offset** tool to set off construction lines at the same distances dP and dAB from the Secondary Auxiliary View Ref Plane line (See *Figure 1C-4E*).

Next, cross-referencing between the Top view and the Primary Auxiliary view, determine the location of points P, A and B in the Secondary Auxiliary view; then use the **Point** tool to create point P and point A,B (the point end view of the line A-B) in the Secondary Auxiliary view. Finally, use the **Aligned Dimension** tool to measure the shortest distance between point P and line A-B, in the Secondary Auxiliary view (*Figure 1C-4E*).
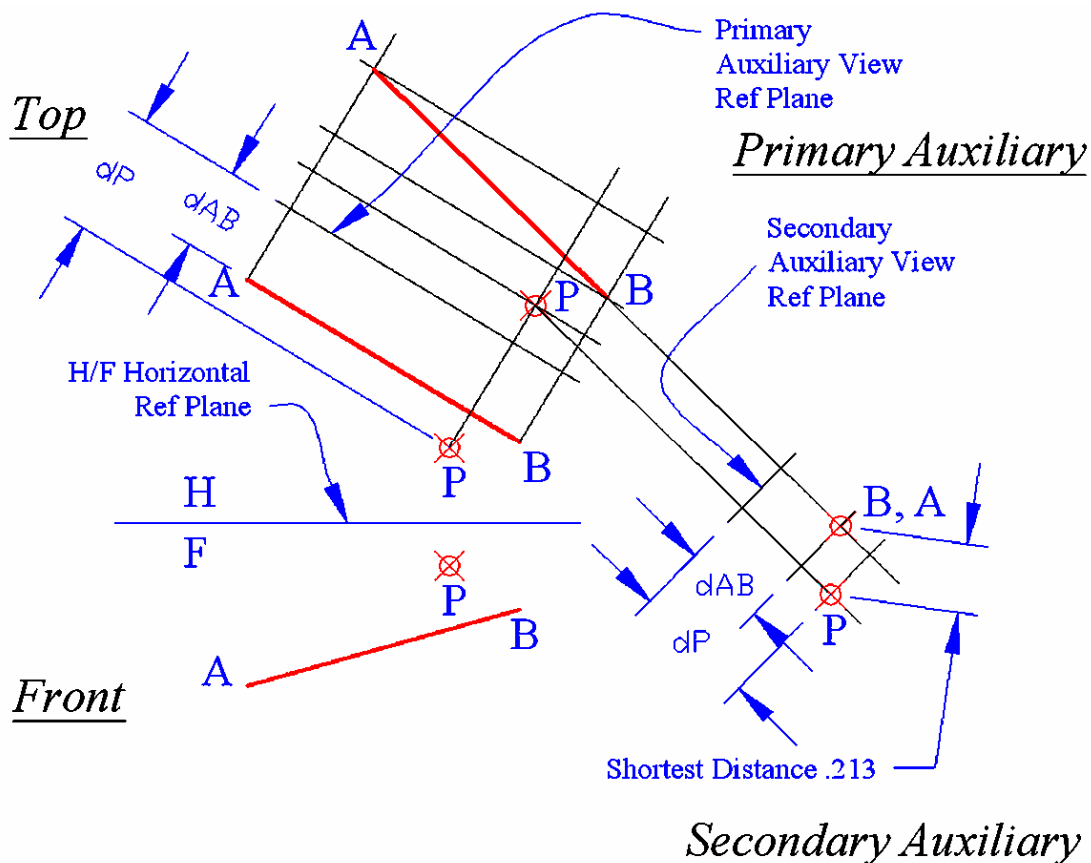
Save the file.

Figure 1C-4E: Obtaining the second set of dimensions in the Top view, setting off the same dimensions in the Secondary Auxiliary view; and creating the point view of the line A-B.

Section 5
Determining the Shortest Distance between a Line and a Point
(Given: Two Adjacent Views, One Showing the Line at True-length)

To start, in the *Points, Lines, & Planes in the Three-Dimensional Space.dwg* file (inside the *Given Problems* folder, in the *Student Descriptive Geometry Learning Resource CD*); use the **Zoom** tool to zoom in the area under the tile of *Determining the Shortest Distance between a Line and a Point (Given: Two Adjacent Views, One Showing the Line at True-length)*; and keep the **ORTHO** and **OTRACK** buttons beneath the **Command Line** turned on, and the **Node** running **Object Snap** in the **Drafting Settings** window turned on.

In this *Section*, two adjacent views of a line and a point are given; and in the Front view, the line A-B is shown at true-length, since in the Top view, the line is shown parallel to the horizontal H/F plane folding line. The conditions given in the problem are different from those given in *Section 4*, since the true-length view of the line is already given (*Figure 1C-5A*).
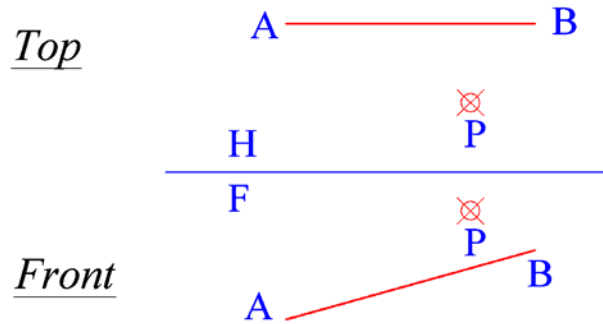

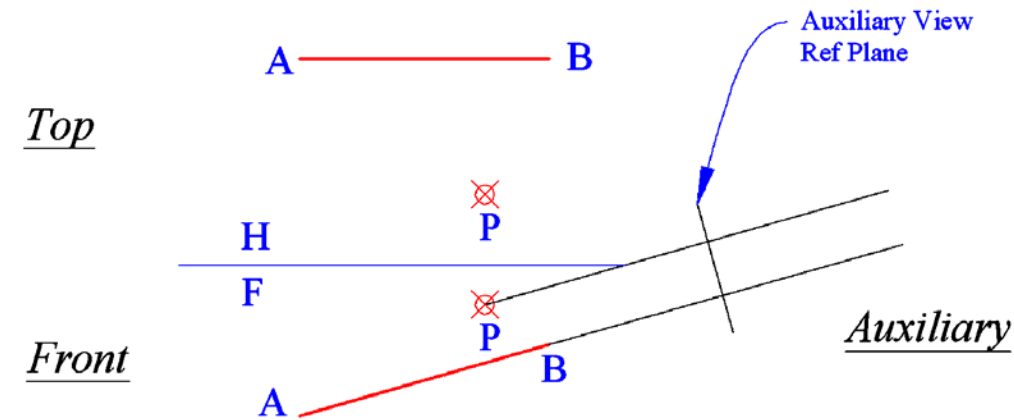*Figure 1C-5A: The given true-length view of the line on the Top view.*

Figure 1C-5B: Projecting the Auxiliary view.

Creating the Auxiliary view (the point view of the line):

To solve this problem, an auxiliary view is first projected upon the Front view (*Figure 1C-5B*) to find the point view of the line, and the shortest distance between the line and the point is measured in the Auxiliary view. First, align the **UCS** with line A-B in the Front view; turn **ORTHO** on; use the **Line** tool to create projection lines from the points A, B, and P, as well as the Auxiliary View Ref Plane line; then return **UCS** to **World** settings.
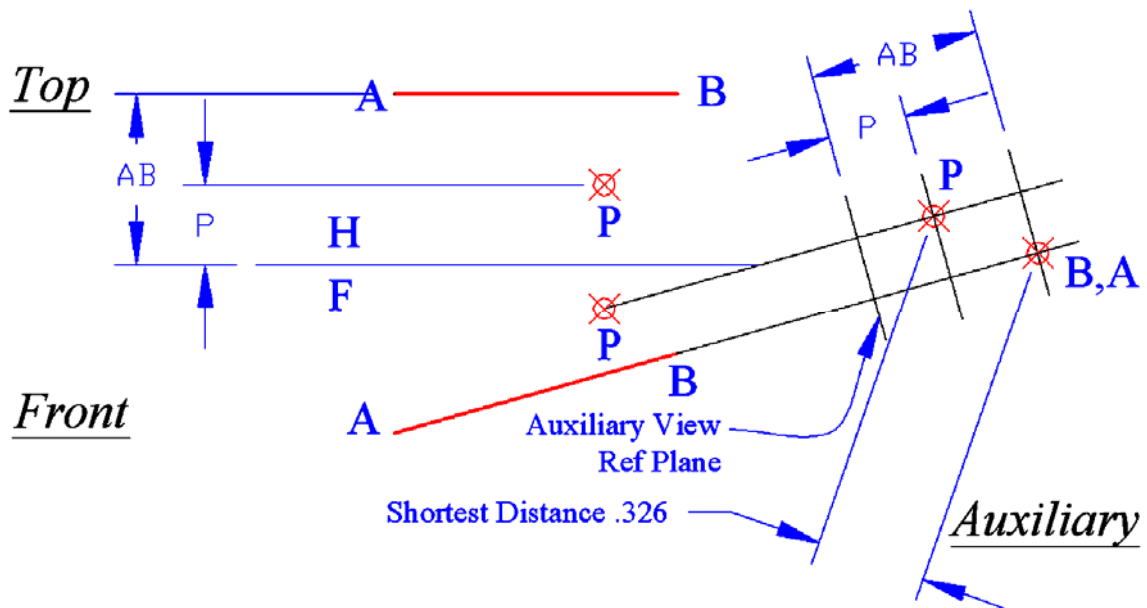

Figure 1C-5C: Creating the point P and the point view of line A-B, and determining the shortest distance between point P and line A-B.

Next, use the **Linear Dimensions** tool to measure the distances from the horizontal H/F plane folding line to point P and line A-B in the Top view; then use the **Offset** tool to set off construction lines at same distances from the Auxiliary View Ref Plane line.

Determining the shortest distance between the given point and line:

Use the **Point** tool to create the point P and the point view of line A-B; and use the **Align Dimension** tool to determine the shortest distance between point P and line A-B, in the Auxiliary view (*Figure 1C-5C*).

This completes the Modules 1C. Congratulations! You have learned how to:

- Determine the shortest distance between two lines, and between a point and a line, using two-dimensional projection method;

- Determine the shortest distance between two lines, using AutoCAD three-dimensional modeling and measuring tools;

- Determine the shortest distance between two lines, with mathematical calculations.